CHAPTER 13

NUCLEI

13.1 ATOMIC NUCLEUS : ITS CONSTITUENTS

1. *Mention the different constituents of an atomic nucleus.*

Atomic nucleus : Its constituents. To explain the large angle scattering of α-particles by thin metal foils, *Rutherford* in 1911 postulated the existence of a nucleus inside an atom.

1. According to Rutherford's planetary model of atom, the entire positive charge and most of the mass of the atom are concentrated in a small volume called the *nucleus* and a suitable number of electrons revolve around it just as planets revolve around the sun.
2. From the results of Rutherford scattering experiments, nuclear size is found to be of the order of 10^{-14} m whereas the diameter of an atom is of the order of 10^{-10} m. Hence most of the atom is empty.
3. The studies of natural radioactivity revealed that the emissions of α-, β- and γ-particles/ radiations have nuclear origin. In a real sense, the γ-rays are not the constituents of nuclei but they are emitted when a nucleus in excited state returns to the ground state.
4. Researches on artificial radioactivity revealed that many particles like α-particles, protons, neutrons, positrons, β-particles, etc. enter into the constitution of the nuclei in one way or the other.
5. Finally, the cosmic ray studies established the existence of new fundamental particles, called mesons, which occur in many forms, having different masses and charges.

13.2 COMPOSITION OF A NUCLEUS

2. *What is proton-neutron hypothesis of nuclear composition ? Define the various terms used to describe nuclear composition.*

Proton-neutron hypothesis of nuclear composition. The discovery of neutrons by *Chadwick*, led *Heisenberg* to propose *proton-neutron hypothesis* in 1932. According to this hypothesis, protons and neutrons are the main building blocks of the nuclei of all atoms. Thus a nucleus of mass number A and atomic number Z contains Z protons and $(A-Z)$ neutrons. The protons give positive charge to the nucleus, while protons and neutrons together give it mass. To neutralise the positive charge of the nucleus, *i.e.*, to make the atom electrically neutral, the number of extra-nuclear electrons is Z.

Proton. It is a fundamental particle which may be called the nucleus of hydrogen. It has a positive charge of 1.6×10^{-19} C. It has a rest mass of 1.6726×10^{-27} kg, which is about 1836 times the rest mass of an electron. A proton has an intrinsic (spin) angular momentum equal to 1/2. It also possesses a magnetic moment much smaller than that of an electron.

Neutron. It is a chargeless fundamental particle having mass slightly greater than that of a proton. Its rest mass is 1.6749×10^{-27} kg. It has intrinsic angular momentum equal to that of a proton. Inspite of being neutral, a neutron also possesses a small magnetic moment.

Neutrons and protons are identical particles in the sense that their masses are nearly the same and the force, called *nuclear force*, does not distinguish them. So the neutrons and protons have common name, the *nucleons*. However, as the proton is positively charged and the neutron is electrically neutral, so the electromagnetic force can distinguish the two types of particles.

The following terms are used to describe the composition of an atomic nucleus :

1. **Nucleons.** *Protons and neutrons which are present in the nuclei of atoms are collectively known as nucleons.*

2. **Atomic number.** *The number of protons in the nucleus is called the atomic number of the element.* It is denoted by Z.

3. **Mass number.** *The total number of protons and neutrons present in a nucleus is called the mass number of the element.* It is denoted by *A*.

Hence *for a neutral atom*, we have the following relations :

Number of protons in an atom	$= Z$
Number of electrons in an atom	$= Z$
Number of nucleons in an atom	$= A$
Number of neutrons in an atom	$= N = A - Z$.

4. **Nuclear mass.** *The total mass of the protons and neutrons present in a nucleus is called the nuclear mass.*

5. **Nuclide.** When an atom is talked of with particular reference to its nuclear composition, it is called a nuclide. *Thus a nuclide is a specific nucleus of an atom characterised by its atomic number Z and mass number A.*

It is symbolically represented as

$$^{A}_{Z}X$$

where, X = chemical symbol of the element,

Z = atomic number, and

A = mass number.

For example, gold nucleus is represented as $^{197}_{79}Au$. It contains 197 nucleons, of which 79 are protons and 118 neutrons.

13.3 ISOTOPES, ISOBARS, ISOTONES AND ISOMERS

3. *What are isotopes, isobars, isotones and isomers ? Give suitable examples.*

Isotopes. *The atoms of an element which have the same atomic number but different mass number are called isotopes.* Such atoms contain the same number of protons and electrons but different number of neutrons. Because of their similar electronic configuration, isotopes of an element exhibit similar chemical properties and they occupy the same position in the periodic table.

Hydrogen has three isotopes : *Hydrogen (protium)* $^{1}_{1}H$ – its nucleus has just one proton ; *deuterium* ($^{2}_{1}H$) – its nucleus has one proton and one neutron ; and *tritium* ($^{3}_{1}H$) – its nucleus has one proton and two neutrons, as shown in Fig. 13.1.

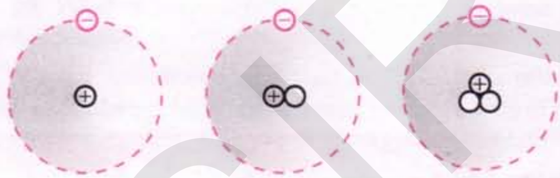

Fig. 13.1 Isotopes of hydrogen.

Lithium has two isotopes $^{6}_{3}Li$ and $^{7}_{3}Li$, as shown in Fig. 13.2.

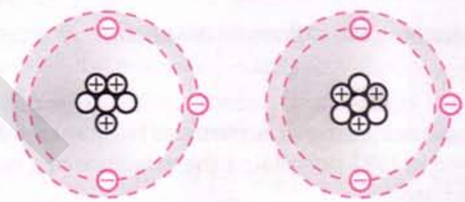

Fig. 13.2 Isotopes of lithium.

Gold has 32 isotopes, ranging from $A = 173$ to 204.

The different isotopes of an element are found to have different relative abundances. So the weighted average of the atomic masses of all the isotopes of an element is taken as its average atomic mass. For example, normal chlorine contains 75% of $^{35}_{17}Cl$ and 25% of $^{37}_{17}Cl$.

∴ Average atomic mass of chlorine

$$= \frac{35 \times 75 + 37 \times 25}{75 + 25} = 35.50$$

Isobars. *The atoms having the same mass number but different atomic number are called isobars.* Such atoms contain different number of protons and electrons. So they differ in the chemical properties and occupy different positions in the periodic table. Some examples of isobars are :

1. $^{3}_{1}H$ and $^{3}_{2}He$, as both have same $A = 3$.

2. $^{37}_{17}Cl$ and $^{37}_{16}S$, as both have same $A = 37$.

3. $^{40}_{20}Ca$ and $^{40}_{18}Ar$, as both have same $A = 40$.

Isotones. *The nuclides having the same number of neutrons are called isotones.* For example,

1. ${}^{37}_{17}Cl$ and ${}^{39}_{19}K$ are isotones, as both contain the same number of neutrons *i.e.*, for both

$$N = A - Z = 20.$$

2. ${}^{198}_{80}Hg$ and ${}^{197}_{79}Pu$ are isotones, as for both

$$N = A - Z = 118.$$

Isomers. *These are the nuclei with same atomic number and same mass number but existing in different energy states.* For example, a nucleus in its ground state and the identical nucleus in metastable excited state are isomers.

13.4 ATOMIC MASSES

4. *Define the terms atomic mass unit and electron volt. Express atomic mass unit in terms of MeV.*

Atomic mass unit. The mass of the carbon-12 atom is 1.992678×10^{-26} kg, which is very small. Therefore, it is useful to choose a convenient unit for expressing the mass of atoms. This unit is defined by taking mass of carbon-12 atom equal to 12 atomic mass units.

One atomic mass unit is defined as $\frac{1}{12}$th of the actual mass of carbon-12 atom.

Atomic mass unit is denoted by *amu* or just by *u*.

Thus

$$1 \text{ amu} = \frac{1}{12} \times \text{Mass of carbon-12 atom}$$

$$= \frac{1}{12} \times 1.992678 \times 10^{-26} \text{ kg}$$

or $$\mathbf{1 \text{ amu} = 1.660565 \times 10^{-27} \text{ kg}}$$

We can now express different masses in terms of amu.

Mass of an electron,

$$m_e = 0.00055 \text{ amu} = 9.11 \times 10^{-31} \text{ kg}$$

Mass of a proton,

$$m_p = 1.0073 \text{ amu} = 1.6726 \times 10^{-27} \text{ kg}$$

Mass of a neutron,

$$m_n = 1.0086 \text{ amu} = 1.6749 \times 10^{-27} \text{ kg}$$

Mass of a hydrogen atom,

$$m_H = m_p + m_e = 1.0078 \text{ amu}$$

The atomic masses can be measured accurately by using an instrument called mass spectrometer.

Electron Volt. *It is defined as the energy acquired by an electron when it is accelerated through a potential difference of 1 volt and is denoted by eV.*

$$\mathbf{1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}}$$

It is a convenient unit of energy used commonly in atomic physics. For example, 13.6 eV energy is needed to remove an electron from a hydrogen atom. A bigger unit called *million electron volt* (MeV) is used for measuring energy changes in nuclear reactions. For example, about 2.2 MeV energy is needed to separate neutron and proton in a deuterium nucleus.

$$\mathbf{1 \text{ MeV} = 10^6 \text{ eV} = 1.602 \times 10^{-13} \text{ J}}$$

Relation between amu and MeV : Energy equivalent of amu. The Einstein's *mass-energy equivalence* relation is

$$E = mc^2$$

This relation shows that the *energy content of an object is equal to its mass times the square of the speed of light*. To determine the energy equivalent of one atomic mass unit, we take

$$m = 1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$$

$$c = 2.998 \times 10^8 \text{ ms}^{-1}$$

Then $$E = 1.66 \times 10^{-27} \times (2.998 \times 10^8)^2 \text{ J}$$

$$= \frac{1.66 \times 10^{-27} \times (2.998 \times 10^8)^2}{1.602 \times 10^{-19}} \text{ eV}$$

$$\simeq 931 \text{ MeV}$$

$$\therefore \quad \mathbf{1 \text{ amu} = 931 \text{ MeV}.}$$

13.5 NUCLEAR SIZE

5. *How is the size of a nucleus estimated ? Write the relation between the radius of a nucleus and its mass number.*

Nuclear size. Like an atom, a nucleus is not a solid object. Its surface is not a well-defined boundary. Still we can assign a size to the nucleus.

By performing scattering experiments using high energy probes such as fast moving protons, neutrons or electrons, nuclear sizes of different elements have been accurately measured. Assuming nuclei to be spherical, their volumes can be estimated.

Experimental observations show that *the volume of a nucleus is directly proportional to its mass number.*

If *R* is the radius of a nucleus having mass number *A*, then

$$\frac{4}{3}\pi R^3 \propto A$$

or $$R \propto A^{1/3}$$

Thus, *the radius R of a nucleus is proportional to cube root of its mass number*. We can write

$$R = R_0 A^{1/3}$$

Here R_0 is a constant, which is of the order of the range of nuclear force. It is believed to be the average nucleon size and is known as *nuclear unit radius*. The value of R_0 depends on the nature of probe particles. For electrons,

$$R_0 = 1.2 \times 10^{-15} \text{m} = 1.2 \text{ fm}$$

13.6 NUCLEAR DENSITY

6. *Prove that the nuclear density is same for all nuclei. Give an estimate of nuclear density.*

Nuclear density. The density of nuclear matter is the ratio of the mass of a nucleus to its volume. As the volume of a nucleus is directly proportional to its mass number A, *so the density of nuclear matter is independent of the size of the nucleus*. Thus the nuclear matter behaves like a liquid of constant density. Different nuclei are like drops of this liquid, of different sizes but of same density.

Let A be the mass number and R be the radius of a nucleus. If m is the average mass of a nucleon, then

Mass of nucleus $= mA$

Volume of nucleus

$$= \frac{4}{3}\pi R^3$$

$$= \frac{4}{3}\pi (R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

$\therefore$ Nuclear density,

$$\rho_{nu} = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}}$$

or $$\rho_{nu} = \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Clearly, *nuclear density is independent of mass number A or the size of the nucleus.*

Taking $m = 1.67 \times 10^{-27}$ kg

and $R_0 = 1.2 \times 10^{-15}$ m, we get

$$\rho_{nu} = \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.142 \times (1.2 \times 10^{-15})^3}$$

$$= 2.30 \times 10^{17} \text{ kg m}^{-3}$$

Thus the nuclear mass density is of the order 10^{17} kg m^{-3}. This density is very large as compared to the density of ordinary matter, say water, for which $\rho = 1.0 \times 10^3$ kg m^{-3}.

For Your Knowledge

- The density of nuclei of all elements is same, independent of the mass number A. The mass density of atom does not follow this rule.
- The density of nuclear matter ($\approx 10^{17}$ kg m^{-3}) is extremely large compared to the average density of matter distributed in the atom. Such a high density exists inside the *neutron stars* whose inside temperature is so large that the atoms have been completely stripped of their orbital electrons.
- The nuclear density is not uniform throughout the nucleus. It has a maximum value at the centre and gradually decreases to zero as we move away from the centre. The effective value of nuclear radius is taken as the distance from the centre, at which the density decreases to half of its value at the centre.

Examples based on
Equivalent Energy, Atomic Mass, Nuclear Size and Nuclear Density

Formulae Used

1. Einstein's mass-energy equivalence, $E = mc^2$
2. $1 \text{ amu} = \frac{1}{12} \times \text{Mass of C-12 atom}$
3. Nuclear radius, $R = R_0 A^{1/3}$
   where $R_0 = 1.2 \times 10^{-15}$ m
4. $\rho_{nu} = \frac{\text{Nuclear mass}}{\text{Nuclear volume}} = \frac{m_{nu}}{\frac{4}{3}\pi R^3}$
5. Average atomic mass of an element
   = Weighted average of the masses of all isotopes.

Units Used

R and R_0 are in metre, ρ in kgm^{-3}.

Conversions Used

1 amu $= 1.66 \times 10^{-27}$ kg $= 931$ MeV,
1 eV $= 1.6 \times 10^{-19}$ J.

Example 1. *Express 16 mg mass into equivalent energy in eV.* [Punjab 2000]

Solution. Here $m = 16 \text{ mg} = 16 \times 10^{-6}$ kg,

$$c = 3 \times 10^8 \text{ms}^{-1}$$

$\therefore$ Equivalent energy,

$$E = mc^2 = 16 \times 10^{-6} \times (3 \times 10^8)^2 \text{ J}$$

$$= \frac{16 \times 10^{-6} \times (3 \times 10^8)^2}{1.6 \times 10^{-19}} \text{ eV} = \mathbf{9 \times 10^{30} \text{ eV}}.$$

Example 2. *How many electron volts make up one joule ?* [Himachal 93]

Solution. By definition, one electron volt is the energy gained by an electron, when accelerated through a potential difference of 1 volt, therefore

$$1\text{ eV} = 1.602 \times 10^{-19}\text{C} \times 1\text{V} = 1.602 \times 10^{-19}\text{ J}$$

Hence $$1\text{ J} = \frac{1}{1.602 \times 10^{-19}}\text{ eV} = \mathbf{6.242 \times 10^{18}\ eV.}$$

Example 3. *Taking one atomic mass unit equal to 931 MeV, calculate the mass of $^{12}_{6}C$ atom.*

Solution. Given 1 amu = 931 MeV

$$= 931 \times 1.602 \times 10^{-13}\text{ J}$$

Using Einstein's mass-energy relationship,

$$E = mc^2 \quad \text{or} \quad m = \frac{E}{c^2}$$

$$\therefore \quad 1\text{ amu} = \frac{931 \times 1.602 \times 10^{-13}}{(3 \times 10^8)^2}\text{ kg}$$

$$= 1.657 \times 10^{-27}\text{ kg}$$

$\therefore$ Mass of $^{12}_{6}C$ atom

$$= 12\text{ amu} = 12 \times 1.657 \times 10^{-27}\text{ kg}$$

$$= \mathbf{1.988 \times 10^{-26}\ kg.}$$

Example 4. *The natural chlorine is found to be a mixture of two isotopes of masses 34.98 amu and 36.98 amu respectively. Their relative abundances are 75.4 and 24.6 percent respectively. Find the composite atomic mass of natural chlorine.*

Solution. The average atomic mass of chlorine is

$$m(\text{Cl}) = \frac{75.4 \times 34.98 + 24.6 \times 36.98}{100}\text{ amu}$$

$$= \frac{2637.49 + 909.71}{100}\text{ amu} = \mathbf{35.47\ amu.}$$

Example 5. *The natural boron is composed of two isotopes of $^{10}_{5}B$ and $^{11}_{5}B$. The two isotopes have masses 10.003 amu and 11.009 amu, respectively. Find the relative abundance of each isotope in the natural boron if the atomic mass of natural boron is 10.81 amu.*

Solution. Suppose the natural boron contains x% of $^{10}_{5}B$ isotope and $(100 - x)$% of $^{11}_{5}B$ isotope.

Then,

Atomic mass of natural boron = Weighted average of the masses of two isotopes

$$\therefore \quad 10.81 = \frac{x \times 10.003 + (100 - x) \times 11.009}{100}$$

or $$1081 = -1.006\,x + 1100.9$$

or $$x = \frac{19.9}{1.006} = 19.78$$

$\therefore$ Relative abundance of $^{10}_{5}B$ isotope = **19.78%**

Relative abundance of $^{11}_{5}B$ isotope = **80.22%.**

Example 6. *Calculate the radius of Ge^{70}. Given $R_0 = 1.1$ fm*

Solution. Here $A = 70$, $R_0 = 1.1$ fm

$$R = R_0\, A^{1/3} = 1.1 \times (70)^{1/3} = 1.1 \times 4.12 = \mathbf{4.53\ fm.}$$

Example 7. *The nuclear mass of $^{56}_{26}Fe$ is 55.85 amu. Calculate its nuclear density.*

Solution. Here

$$m_{\text{Fe}} = 55.85\text{ amu} = 55.85 \times 1.66 \times 10^{-27}\text{ kg}$$
$$= 9.27 \times 10^{-26}\text{ kg}$$

Nuclear radius $= R_0\, A^{1/3} = 1.1 \times 10^{-15} \times (56)^{1/3}$ m [$\because A = 56$]

$$\rho_{nu} = \frac{\text{Nuclear mass}}{\text{Nuclear volume}} = \frac{m_{\text{Fe}}}{\frac{4}{3}\pi R^3}$$

$$= \frac{9.27 \times 10^{-26}}{\frac{4\pi}{3} \times (1.1 \times 10^{-15})^3 \times 56}$$

$$= \mathbf{2.9 \times 10^{17}\ kg\ m^{-3}.}$$

Example 8. *Assuming that protons and neutrons have equal masses, calculate how many times nuclear matter is denser than water. Given that nuclear radius is given by $R = 1.2 \times 10^{-15} A^{1/3}$ metre and mass of a nucleon $= 1.67 \times 10^{-27}$ kg.*

Solution. Here $R = 1.2 \times 10^{-15} A^{1/3}$ m,

Mass of a nucleon, $m = 1.67 \times 10^{-27}$ kg

Nuclear mass $= mA = 1.67 \times 10^{-27} \times A$ kg

Nuclear volume $= \frac{4}{3}\pi R^3$

$$= \frac{4}{3}\pi (1.2 \times 10^{-15} A^{1/3})^3\text{ m}^3$$

Nuclear density,

$$\rho_{nu} = \frac{1.67 \times 10^{-27} \times A}{\frac{4}{3}\pi (1.2 \times 10^{-15} A^{1/3})^3}$$

$$= 2.307 \times 10^{17}\text{ kg m}^{-3}$$

Now, density of water,

$$\rho_{wat} = 10^3\text{ kg m}^{-3}$$

$$\therefore \quad \frac{\rho_{nu}}{\rho_{wat}} = \frac{2.307 \times 10^{17}}{10^3} = \mathbf{2.307 \times 10^{14}.}$$

Problems For Practice

1. What is equivalent energy of a 10 mg mass ? **[CBSE D 92]**

(Ans. 9×10^9 J)

2. Calculate the mass equivalent of 1 amu. (Ans. 1.66×10^{-27} kg)

3. Express 1 electron volt in kilowatt hour. (Ans. 1 eV $= 4.4 \times 10^{-26}$ kWh)

4. Find the effective mass of a photon of energy 5 eV. (Ans. 8.889×10^{-36} kg)

5. Find the effective mass of a photon if the frequency of the radiation is 6×10^{14} Hz. (Ans. 4.42×10^{-36} kg)

6. Find the effective mass of a photon if the wavelength of the radiation is 3000 Å. (Ans. 7.367×10^{-36} kg)

7. What is the order of energy, in eV for a photon of visible light ? (Ans. 2.1 eV)

8. Calculate the nuclear radii of ${}^{140}_{56}Ba$ and ${}^{17}_{8}O$. Given $R_0 = 1.5$ fm. (Ans. 7.888 fm, 3.857 fm)

9. The nuclear radius of Pb^{208} is 8.874 fm. What will be the nuclear radius of Ca^{44} ? (Ans. 5.286 fm)

10. A nucleus with $A = 235$ splits into two nuclei whose mass numbers are in the ratio 2 : 1. If $R_0 = 1.4$ fm, find the radii of the new nuclei. (Ans. 5.99 fm, 7.55 fm)

11. Calculate the density of hydrogen nucleus in SI units. Given $R_0 = 1.1$ fermi and $m_p = 1.007825$ amu. (Ans. 2.98×10^{17} kgm^{-3})

12. The nuclear radius of ${}^{16}_{8}O$ is 3×10^{-15} m. Find the density of nuclear matter. (Ans. 2.359×10^{17} kgm^{-3})

13. Find nuclear mass density of ${}^{238}_{92}U$. Given $R_0 = 1.5$ fermi and mass of each nucleon $= 1.67 \times 10^{-27}$ kg. (Ans. 1.18×10^{17} kgm^{-3})

HINTS

2. Mass of 6.023×10^{23} carbon atoms = 12 g

$$\therefore \quad 1\,\text{amu} = \frac{1}{12} \times \text{Mass of carbon-12 atom}$$

$$= \frac{1}{12} \times \frac{12}{6.023 \times 10^{23}} = \mathbf{1.66 \times 10^{-27}\ kg.}$$

3. $$1\,\text{eV} = 1.6 \times 10^{-19}\ \text{J} = \frac{1.6 \times 10^{-19}}{3.6 \times 10^{6}}\ \text{kWh}$$

$$= \mathbf{4.4 \times 10^{-26}\ kWh.}$$

4. $$m = \frac{E}{c^2} = \frac{5 \times 1.6 \times 10^{-19}}{(3 \times 10^{8})^2} = \frac{8}{9} \times 10^{-35}$$

$$= \mathbf{8.889 \times 10^{-36}\ kg.}$$

5. $$m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{6.63 \times 10^{-34} \times 6 \times 10^{14}}{(3 \times 10^{8})^2}$$

$$= \mathbf{4.42 \times 10^{-36}\ kg.}$$

6. $$m = \frac{E}{c^2} = \frac{hc}{c^2\lambda} = \frac{h}{c\lambda} = \frac{6.63 \times 10^{-34}}{3 \times 10^{8} \times 3 \times 10^{-7}}$$

$$= \mathbf{7.367 \times 10^{-36}\ kg.}$$

7. Take mean wavelength for yellow light = 5893 Å.

11. $$\rho = \frac{3\,m_p}{4\pi\,R_0^3} = \frac{3 \times 1.007825 \times 1.66 \times 10^{-27}}{4 \times \frac{22}{7} \times (1.1 \times 10^{-15})^3}$$

$$= \mathbf{2.98 \times 10^{17}\ kg\ m^{-3}.}$$

12. Here $R = 3 \times 10^{-15}$ m

Nuclear mass = 16 amu $= 16 \times 1.66 \times 10^{-27}$ kg

$$\rho_{nu} = \frac{\text{Nuclear mass}}{\text{Nuclear volume}} = \frac{16 \times 1.66 \times 10^{-27}}{\frac{4}{3}\pi(3 \times 10^{-15})^3}$$

$$= \mathbf{2.359 \times 10^{17}\ kg\ m^{-3}.}$$

13.7 DISCOVERY OF NEUTRONS

7. *Briefly explain how neutrons were discovered. Give some important properties of neutrons.*

Discovery of neutrons. The neutrons were discovered by *James Chadwick* in 1932. He was awarded the 1935 Nobel prize for physics for this discovery.

In 1932, Chadwick performed an experiment in which α-particles from a radioactive Polonium source were used to bombard beryllium nuclei. Highly penetrating rays were found to come out of the

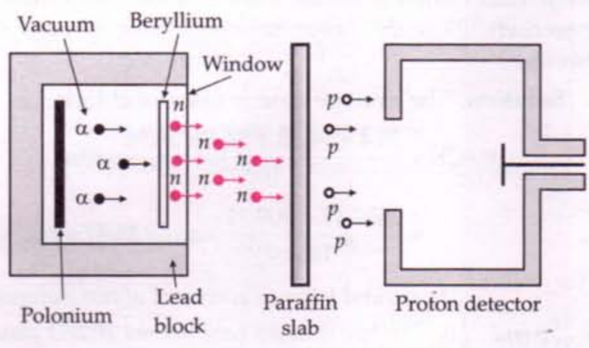

Fig. 13.3 Experimental set up used by Chadwick to discover neutrons.

beryllium metal, which could not be deflected by electric and magnetic fields. These radiations were used to bombard hydrocarbons like paraffin wax. High energy protons were knocked out from the paraffin wax. The energy of the ejected protons was found to be too high to be accounted for γ-ray photons. By using the laws of conservation of energy and momentum, Chadwick concluded that the penetrating radiation consisted of *neutral particles, each having a mass nearly that of a proton*. These particles were called *neutrons*.

The reaction may be written as

$$_2^4\text{He} + _4^9\text{Be} \longrightarrow _0^1n + _6^{12}\text{C}$$

Here $_0^1n$ denotes a *neutron* having zero charge and mass nearly the same as that of a proton.

Properties of neutrons :

1. Neutron is an elementary particle present in the nuclei of all elements except hydrogen.
2. Neutron has no charge and its mass is slightly more than that of a proton
$$m_n = 1.00866 \text{ amu} = 1.6749 \times 10^{-27} \text{kg}$$
3. Inside a nucleus, a neutron is stable. But outside a nucleus, it is unstable. A free neutron spontaneously decays into a proton, electron and antineutron (an elementary particle with zero charge and zero rest mass) with a mean life of about 1000 s.
$$_0^1n \rightarrow _1^1\text{H} + _{-1}^{0}e + \bar{\nu} + Q$$
4. Being neutral, they do not interact with electrons. So neutrons have low ionising powers.
5. Being neutral, neutrons are not repelled or attracted by the nucleus and the electrons of an atom. They can easily penetrate heavy nuclei and induce nuclear reactions.
6. They induce radioactivity in many elements.
7. In heavier nuclei, the number of neutrons is more than that of protons. Protons being positively charged, repel each other and in order to maintain stability of the nucleus, more neutrons become necessary for heavier nuclei.

13.8 NUCLEAR FORCE

8. *What are nuclear forces ? Give their important properties.*

Nuclear force. The average separation between two nucleons is about 10^{-15} m. At this separation, positively charged protons feel strong coulombic repulsion. Also the gravitational force of attraction between two nucleons is about 10^{-36} times smaller than the electrostatic repulsion, it cannot hold the nucleons together. So there must be some other strong attractive force acting between the nucleons that over-comes the electrostatic repulsion. *This strong attractive interaction acting between the nucleons is called* ***nuclear force*** or ***strong interaction***.

Nuclear force is a strong attractive force that binds the protons and neutrons together inside a tiny nucleus.

Properties of nuclear force :

1. **Strongest interaction.** Nuclear force is the strongest interaction known in nature that holds the nucleons together despite the strong electrostatic repulsion between the protons. The relative strength of gravitational, electrostatic and nuclear forces is

$$F_g : F_e : F_n = 1 : 10^{36} : 10^{38}$$

2. **Short-range force.** Unlike gravitational and electrostatic forces, nuclear force is a short-range force. It operates only upto a very short distance of about 2-3 fm from a nucleon.

3. **Variation with distance.** The graph of P.E. of a pair of nucleons as a function of their separation r is shown in Fig. 13.4. The P.E. is minimum at a distance $r_0 \simeq 0.8$ fm.

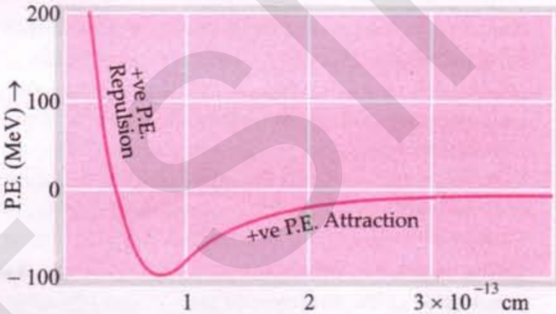

Fig. 13.4 Graph of P.E. a pair of nucleons as a function of their separation.

(*i*) For $r < r_0$, the P.E. increases rapidly with decreasing r. It indicates a strong repulsive nuclear force.

(*ii*) For $r > r_0$, the P.E. gradually decreases to zero with increasing r. It indicates attractive nuclear force. The attraction is maximum at $r_0 \simeq 0.8$ fm and varies inversely not as the square of distance but depends on some higher power of distance.

(*iii*) For $r \simeq 4$ fm, the nuclear force becomes zero. It indicates that nuclear force is a short range force.

The negative sign of P.E. signifies that the nuclear force is attractive.

4. **Charge independent character.** It is seen from experiments that the attractive force between two neutrons (*nn-force*) is nearly equal to that between two protons (*pp-force*) or between a proton and a neutron (*pn-force*). Thus the nuclear force does not depend on the charge of the particles.

In case of *pp-nuclear force,* there is a repulsive force between two protons, but this is weak compared to the strong nuclear force.

5. **Saturation effect.** Nuclear forces show saturation effect, *i.e.,* a nucleon interacts only with its neighbouring nucleon. This property is supported by the fact that the binding energy per nucleon is same over a wide range of mass numbers.

6. **Spin dependent character.** The nuclear force between two nucleons having parallel spins is stronger than that between two nucleons having antiparallel spins.

7. **Exchange forces.** In 1935, a Japanese physicist *H.Yukawa* suggested that the nuclear force between two nucleons arises from the constant exchange of particles, called *mesons,* between them.

8. **Non-central forces.** The nuclear force between two nucleons does not act along the line joining their centres.

For Your Knowledge

- According to the present view, the nuclear force that binds protons and neutrons together in a nucleus is not a true fundamental force of nature. The nucleons themselves are built of subunits called quarks. Thus the nuclear force is a secondary or 'spill over' effect of the strong force that binds the quarks together to form neutrons and protons. This quark-quark force is caused by the exchange of massless particles called *glucons.* Instead of protons or neutrons, now quarks are regarded as the fundamental constituents of matter. There are six different kinds of quarks, but they cannot be released in the same way as the electrons from atoms or neutrons and protons from the nuclei. No one has ever seen a free quark. In interactions involving protons and neutrons of energy many GeV (1 GeV $=10^9$ eV), the quarks go around as quark – antiquark pairs or a combination of three quarks.

13.9 MASS DEFECT AND PACKING FRACTION

9. *What is Einstein's mass-energy equivalence ? What is its importance ? State the law of conservation of mass-energy.*

Einstein's mass-energy equivalence. In his special theory of relativity, Einstein showed that $E = mc^2$

Here c is the speed of light in vacuum and is equal to 3×10^8 ms^{-1}. The above equation expresses equivalence between mass and energy. This equation suggests that even when a particle is at rest (having zero kinetic energy), it still possesses an enormous amount of energy because of its mass. *The mass of a particle measured in a frame of reference in which the particle is at rest, is called* ***rest mass*** and is denoted by m_0. Thus the total energy of a particle is sum of (*i*) its rest mass energy m_0c^2 and (*ii*) its kinetic energy T. That is $E = m_0c^2 + T$

Clearly, the mass of a particle is greater when it is in motion than when it is at rest.

As mass and energy are convertible into each other, we cannot define separate laws of conservation of mass and conservation of energy. We need to define a unified law of conservation of mass and energy together.

The law of conservation of mass-energy *states that the sum of the mass-energy of a system of particles is the same before and after their interaction.*

The most convincing evidence that this principle operates in nature comes from nuclear physics. It is central to our understanding of nuclear energy and harnessing it as a source of power. Using the principle, the Q-value of a nuclear process (decay or reaction) can be expressed also in terms of initial and final masses.

10. *What is mass defect of a nucleus ? Express it mathematically.*

Mass defect. It is found that the mass of a stable nucleus is always less than the sum of the masses of its constituent protons and neutrons in their free state.

The difference between the rest mass of a nucleus and the sum of the rest masses of its constituent nucleons is called its **mass defect**.

Consider the nucleus ${}^A_Z$X. It has Z protons and $(A-Z)$ neutrons. Therefore, its mass defect will be

$$\Delta m = Zm_p + (A-Z)m_n - m$$

where m_p, m_n and m are the rest masses of a proton, neutron and the nucleus ${}^A_Z$X respectively.

11. *What is packing fraction of a nucleus ? Give its physical significance in relation to nuclear stability.*

Packing fraction. *The packing fraction of a nucleus is its mass defect per nucleon.* Thus

$$\text{P.F. of a nucleus} = \frac{\text{Mass defect}}{\text{Mass number}} = \frac{\Delta m}{A}$$

If P.F. is *positive* (as in case of nuclei with mass number less than 20 and above 200), then the nucleus is *unstable.* If P.F. is *negative* (as in case of nuclei with mass number between 20 and 200), then it indicates that some mass has been converted into energy which binds the nucleons together and so the nucleus is *stable. Thus the P.F. is directly related to the availability of nuclear energy and the stability of the nucleus.*

13.10 BINDING ENERGY AND BINDING ENERGY PER NUCLEON

12. *Explain the term 'binding energy' of a nucleus. Derive an expression for it. State clearly the approximation involved. Also write an expression for the binding energy per nucleon.*

Binding energy. An atomic nucleus is a stable structure. Inside it, the protons and neutrons are bound together by means of strong attractive nuclear forces. Thus a definite amount of work is required to be done to break up the nucleus into its constituent particles and to place them at infinite distance from one another. This work gives a measure of the binding energy of the nucleus. Thus,

*The **binding energy** of a nucleus may be defined as the energy required to break up a nucleus into its constituent protons and neutrons and to separate them to such a large distance that they may not interact with each other.*

The concept of binding energy may also be understood in terms of *Einstein's mass energy equivalence*. It is seen that the mass of a stable nucleus is always less than the sum of the masses of the constituent protons and neutrons in their free state. This mass difference is called *mass defect which accounts for the* ΔE_b energy released when a certain number of neutrons and protons are brought together to form a nucleus of a certain charge and mass. Thus

$$\Delta E_b = \Delta m \times c^2$$

So *the binding energy may also be defined as the surplus energy which the nucleons give up by virtue of their attractions when they become bound together to form a nucleus.*

The energy equivalent to the mass defect is radiated in the form of electromagnetic radiation when the nucleons combine to form a nucleus.

Expression for binding energy. The nucleus $^A_Z X$ contains Z protons and $(A - Z)$ neutrons. Its mass defect is

$$\Delta m = Zm_p + (A - Z)m_n - m_N \quad ...(1)$$

where m_N is the nuclear mass of $^A_Z X$. From Einstein's mass-energy equivalence, the binding energy of the nucleus is

$$\Delta E_b = \Delta m \times c^2 = [Zm_p + (A - Z)m_n - m_N]\, c^2 \quad ...(2)$$

Now, in an atom the electrons are bound to the nucleus by electrostatic forces. So they have a binding energy of their own, which from the mass-energy equivalence is given by

$$(\Delta E_b)_e = [(m_N + Zm_e) - m(^A_Z X)]\, c^2 \quad ...(3)$$

where $m(^A_Z X)$ is the atomic mass. The binding energy of electrons ($\approx$ eV to keV) is negligible compared to the binding energy of nucleons ($\approx 10^3$ MeV). It will be a safe approximation to take,

$$(\Delta E_b)_e = 0$$

$$\therefore \quad m_N + Zm_e - m(^A_Z X) = 0$$

or

$$m_N = m(^A_Z X) - Zm_e$$

Thus, in terms of atomic mass the equation (2) becomes

$$\Delta E_b = [Zm_p + (A - Z)m_n - m(^A_Z X) + Zm_e]\, c^2$$

$$= [Z(m_p + m_e) + (A - Z)m_n - m(^A_Z X)]\, c^2 \quad ...(4)$$

But $m_p + m_e = m_H$ = mass of a hydrogen atom.

$\therefore$ The equation (4) can be written in terms of m_H as

$$\Delta E_b = [Zm_H + (A - Z)m_n - m(^A_Z X)]\, c^2 \quad ...(5)$$

Binding energy per nucleon. *The binding energy per nucleon is the average energy required to extract one nucleon from the nucleus. It is obtained by dividing the binding energy of a nucleus by its mass number. The expression for binding energy per nucleon can be written as*

$$\Delta E_{bn} = \frac{\Delta E_b}{A} = \frac{[Zm_H + (A - Z)m_n - m(^A_Z X)]\, c^2}{A} \quad ...(6)$$

The binding energy per nucleon gives a measure of the force which binds the nucleons together inside a nucleus.

13.11 BINDING ENERGY CURVE

13. *Draw a graph showing the variation of binding energy per nucleon with mass number of different nuclei. Give the salient features of the curve. How does this curve explain the release of energy in the processes of nuclear fission and fusion ?*

Binding energy curve. The value of binding energy per nucleon of a nucleus gives a measure of the stability of that nucleus. Greater is the binding energy per nucleon of a nucleus, more stable is the nucleus.

Fig. 13.5 shows the graph of binding energy per nucleon drawn against mass number A.

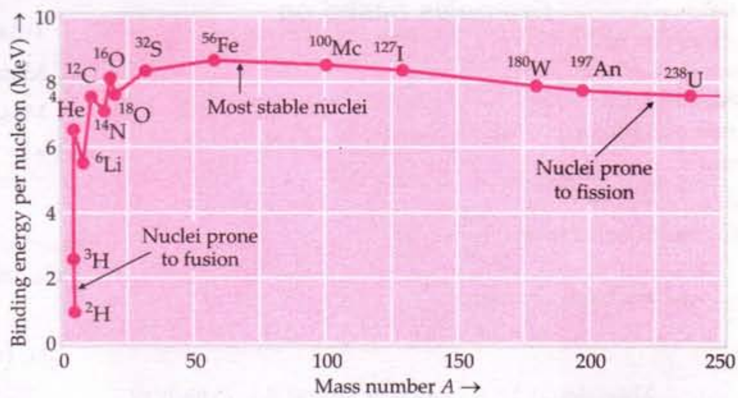

Fig. 13.5 Binding energy per nucleon as a function of mass number A.

The binding energy curve reveals the following important features :

1. Except for some nuclei like $^4_2$He, $^{12}_6$C and $^{16}_8$O, the values of binding energy per nucleon lie on or near a smooth curve.

2. The B.E. /nucleon is small for light nuclei like ${}_1^1H$, ${}_1^2H$ and ${}_1^3H$.

3. In the mass number range 2 to 20, there are well defined maxima and minima on the curve. The maxima occur for ${}_2^4He$, ${}_6^{12}C$ and ${}_8^{16}O$, indicating the higher stability of these nuclei than the neighbouring ones. The minima, corresponding to low stability, occur for ${}_3^6Li$, ${}_5^{10}B$ and ${}_7^{14}N$.

4. The curve has a broad maximum close to the value 8.5 MeV/nucleon in the mass number range from about 40 to 120. It has a peak value of 8.8 MeV/nucleon for ${}_{26}^{56}Fe$.

5. As the mass number increases further, the B.E. / nucleon shows a gradual decrease and drops to 7.6 MeV/nucleon for ${}_{92}^{238}U$. This decrease is due to coulomb repulsion between the protons which makes the heavier nuclei less stable.

Importance of binding energy curve. The binding energy curve can be used to explain the phenomena of nuclear fission and nuclear fusion as follows :

1. **Nuclear fission.** Binding energy per nucleon is smaller for heavier nuclei than the middle ones, *i.e.,* heavier nuclei are less stable. When a heavier nucleus splits into the lighter nuclei, the B.E./nucleon changes from about 7.6 MeV to 8.4 MeV. Greater binding energy of the product nuclei results in the liberation of energy. This is what happens in nuclear fission which is the basis of the atom bomb.

2. **Nuclear fusion.** The binding energy per nucleon is small for light nuclei, *i.e.,* they are less stable. So when two light nuclei combine to form a heavier nucleus, the higher binding energy per nucleon of the latter results in the release of energy. This is what happens in a nuclear fusion which is the basis of the hydrogen bomb.

Examples based on Binding Energy of a Nucleus

Formulae Used

1. Mass defect, $\Delta m = [Z\, m_p + (A - Z)\, m_n - m_N]$
2. B.E. $= (\Delta m)\, c^2$
3. B.E./nucleon $= \dfrac{\text{B.E.}}{A}$
4. Packing fraction $= \dfrac{\Delta m}{A}$.

Units Used

Mass defect Δm is in amu or kg and B.E. in joule or MeV.

Example 9. *Calculate the binding energy of an α-particle in MeV. Given :*

m_p (mass of proton) $= 1.007825$ *amu*
m_n (mass of neutron) $= 1.008665$ *amu*
Mass of He nucleus $= 4.002800$ *amu,*
1 amu $= 931$ *MeV.* [ISCE 98]

Solution. An α-particle contains 2 protons and 2 neutrons.

Mass of 2 protons $= 2 \times 1.007825 = 2.015650$ amu
Mass of 2 neutrons $= 2 \times 1.008665 = 2.017330$ amu
Total mass $= 4.032980$ amu
Mass of He nucleus $= 4.002800$ amu
Mass defect, $\Delta m = 0.030180$ amu
B.E. of α-particle $= 0.030180 \times 931$
$= \mathbf{28.097\ MeV.}$

Example 10. *Express one atomic mass unit in energy units, first in Joules and then in MeV. Using this, express the mass defect of ${}_8^{16}O$ in MeV.* [NCERT]

Solution. We have

$$m = 1\text{ amu} = 1.660565 \times 10^{-27}\text{ kg}$$
$$c = 2.9979 \times 10^8\text{ ms}^{-1}$$
$$\therefore\quad E = mc^2 = 1.660565 \times 10^{-27} \times (2.9979 \times 10^8)^2\text{ J}$$
$$= \mathbf{1.4924 \times 10^{-10}\ J}$$
$$= \frac{1.4924 \times 10^{-10}}{1.602 \times 10^{-13}}\text{ MeV} \qquad [\because 1\text{ MeV} = 1.602 \times 10^{-13}\text{ J}]$$
$$= \mathbf{931.5\ MeV}$$

The ${}_8^{16}O$ nucleus contains 8 protons and 8 neutrons.

Mass of 8 protons $= 8 \times 1.00727 = 8.05816$ amu
Mass of 8 neutrons $= 8 \times 1.00866 = 8.06928$ amu
Total mass $= 16.12744$ amu
Mass of ${}_8^{16}O$ nucleus $= 15.99053$ amu
Mass defect, $\Delta m = 0.13691$ amu

$$\Delta E_b = 0.13691 \times 931.5\text{ MeV}$$
$$= \mathbf{127.5\ MeV.}$$

Example 11. *Calculate the binding energy per nucleon of ${}_{20}^{40}Ca$ nucleus. Given*

$$m({}_{20}^{40}Ca) = 39.962589\ amu$$
m_n (mass of a neutron) $= 1.008665$ *amu*
m_p (mass of a proton) $= 1.007825$ *amu*

[CBSE D 02 ; OD 04]

Solution. The nucleus $^{40}_{20}Ca$ contains 20 protons and 20 neutrons.

Mass of 20 protons
$$= 20 \times 1.007825 = 20.1565 \text{ amu}$$
Mass of 20 neutrons
$$= 20 \times 1.008665 = 20.1733 \text{ amu}$$
Total mass $= 40.3298$ amu

Mass $^{40}_{20}Ca$ nucleus $= 39.962589$ amu

Mass defect, $\Delta m = 0.367211$ amu

$$\text{B.E.} = 0.367211 \times 931 = 341.87 \text{ MeV}$$
$$\text{B.E. per nucleon} = \frac{341.87}{40} = \mathbf{8.547\ MeV}.$$

Problems For Practice

1. The mass of ^7_3Li is 0.042 amu less than the sum of masses of its nucleons. Find the B.E. per nucleon. (Ans. 5.866 MeV)
2. Calculate the binding energy per nucleon for a $^{12}_6C$ nucleus. Atomic mass of $^{12}_6C = 12$ amu, mass of a proton $= 1.007825$ amu, mass of a neutron $=$ 1.008665 amu. (Ans. 7.68 MeV)
3. Calculate the binding energy of a deuteron. Given that
   mass of proton $= 1.007825$ amu
   mass of a neutron $= 1.008665$ amu
   mass of a deuteron $= 2.014103$ amu
   (Ans. 2.22 MeV)
4. The binding energy of $^{20}_{10}Ne$ is 160.6 MeV. Find the atomic mass, given that
   mass of $^1_1H = 1.007825$ amu
   mass of $^1_0n = 1.008665$ amu (Ans. 19.9924 amu)
5. Calculate the binding energy per nucleon (B.E./nucleon) in the nuclei of $^{31}_{15}P$. Given :
   $[^{31}_{15}P] = 30.97376$ amu, $m[^1_0n] = 1.00865$ amu,
   $m[^1_1H] = 1.00782$ amu **[CBSE OD 96 C]**
   (Ans. 8.47 MeV)
6. Calculate the binding energy per nucleon of $^{35}_{17}Cl$ nucleus. Given that
   mass of $^{35}_{17}Cl = 34.980000$ u,
   mass of proton $= 1.007825$ u,
   mass of neutron $= 1.008665$ u and
   1 atomic mass unit (1 u) $= 931$ MeV. **[CBSE OD 02]**
   (Ans. 8.22 MeV]
7. Calculate the binding energy per nucleon of $^{56}_{26}Fe$ nucleus. Given that
   mass of $^{56}_{26}Fe = 55.934939$ amu
   mass of a neutron $= 1.008665$ amu
   mass of a proton $= 1.007825$ amu
   [CBSE D 2000C, 04, 05C] (Ans. 8.79 MeV)
8. Calculate binding energy per nucleon of $^{209}_{83}Bi$. Given that
   $m(^{209}_{83}Bi) = 208.980388$ amu
   m(neutron) $= 1.008665$ amu,
   m(proton) $= 1.007825$ amu. **[CBSE OD 95]**
   (Ans. 7.85 MeV)
9. Calculate the binding energy in MeV of Uranium 238 from the following data :
   Mass of $^1_1H = 1.008142$ amu,
   Mass of $^1_0n = 1.008982$ amu
   Mass of $^{238}_{92}U = 238.124930$ amu
   Also calculate the packing fraction.
   (Ans. 7.5714 MeV, 13.49×10^{-30} kg)

HINTS

1. B.E. of $^7_3Li = \Delta m \times 931 = 0.042 \times 931$ MeV
$$\text{B.E. nucleon} = \frac{0.042 \times 931}{7} = \mathbf{5.866\ MeV}.$$
2. $\Delta m = 6m_p + 6m_n - m(^{12}_6C)$
$$= 6[1.007825 + 1.008665] - 12$$
$$= 12.09894 - 12 = 0.09894 \text{ amu}$$
$$\text{B.E.nucleon} = \frac{0.09894 \times 931}{12} = \mathbf{7.68\ MeV}.$$
3. $\Delta m = m_p + m_n - m(^2_1H)$
$$= 1.007825 + 1.008665 - 2.014103$$
$$= 0.002387 \text{ amu}$$
B.E. of deutron $= 0.002387 \times 931 = \mathbf{2.22\ MeV}.$
5. The nucleus $^{31}_{15}P$ contains 15 protons and 16 neutrons.
   Mass of 15 protons $= 15 \times 1.00782 = 15.1173$ amu
   Mass of 16 neutrons $= 16 \times 1.00865 = 16.1384$ amu
   Total mass $= 31.2557$ amu
   Mass of $^{31}_{15}P$ nucleus $= 30.97376$ amu
   Mass defect, $\Delta m = 0.28194$ amu
   B.E. of $^{31}_{15}P$ nucleus
$$= \Delta m \times 931 \text{ MeV} = 0.28194 \times 931 = 262.486 \text{ MeV}$$
$$\text{B.E./nucleon} = \frac{262.486}{31} = \mathbf{8.47\ MeV}.$$
6. $^{35}_{17}Cl$ has 17 protons and 18 neutrons.
   Mass of 17 protons $= 1.007825 \times 17 = 17.133025$ amu
   Mass of 18 neutrons $= 1.008665 \times 18 = 18.155970$ amu
   Total mass $= 35.288995$ amu
   Mass of $^{35}_{17}Cl$ nucleus $= 34.980000$ amu
   Mass of defect, $\Delta m = 0.308995$ amu

$$B.E. = 0.308995 \times 931 = 287.67 \text{ MeV}$$

$$B.E. \text{ per nucleon} = \frac{287.67}{35}$$

$$= \mathbf{8.22\ MeV.}$$

7. Refer to the answer of Exercise 13.4 on page 13.57.
8. Refer to the answer of Exercise 13.4 on page 13.57.
9. $^{238}_{92}U$ contains 92 protons and 146 neutrons

Mass of 92 protons = 92 × 1.008142 = 92.74064 amu

Mass of 146 neutrons

= 146 × 1.008982 = 147.3137 amu

∴ Total mass = 240.060430 amu

Mass of $^{238}_{92}U$ nucleus = 238.124930 amu

Mass defect, Δm = 1.9355 amu

B.E. of $^{238}_{92}U$ nucleus = 1.9355 × 931 = 1802 MeV

$$\therefore \text{Binding energy/nucleon} = \frac{1802}{238} = \mathbf{7.5714\ MeV}$$

$$\text{Packing fraction} = \frac{\Delta m}{A} = \frac{1.9355}{238} \text{ amu}$$

$$= 8.132 \times 10^{-3} \text{ amu}$$

$$= 8.132 \times 10^{-3} \times 1.66 \times 10^{-27} \text{ kg}$$

$$= \mathbf{13.49 \times 10^{-30}\ kg.}$$

13.12 NUCLEAR ENERGY LEVELS

14. *What do you mean by nuclear energy levels ? What type of radiations are emitted in nucleonic transitions ?*

Nuclear energy levels. The neutrons and protons move inside a nucleus in discrete quantum states with definite energies. These are *nuclear stationary states.* The stationary state of lowest energy is called the *ground state.* If appropriate energy is supplied, a nucleus may be excited from its ground state to the stationary states of higher energy. Fig. 13.6 shows the energy levels of a low mass nuclide, $^{28}_{13}Al$. These energy levels have energy differences of the order of millions of electronvolts (MeV). So when a nucleus makes a transition from some higher energy level to a lower energy level, the difference of energy is emitted as a photon in gamma-ray region of the electromagnetic spectrum.

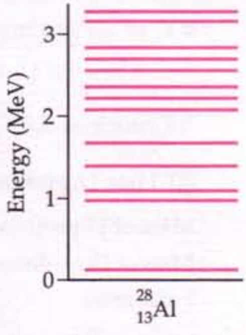

Fig. 13.6 Nuclear energy levels of $^{28}_{13}Al$ nucleus.

13.13 NUCLEAR STABILITY AND UNSTABILITY

15. *How is the stability of a nuclide related to the relative number of neutrons and protons present inside it ? Explain it with the help of the nuclidic chart.*

Nuclear stability and unstability. Some isotopes of an element may be stable while the others may be unstable. A stable nucleus maintains its constitution all the time. An unstable nuclide spontaneously emits a particle and transforms itself into a new nuclide. The stability of a nuclide is intimately connected to the relative number of neutrons and protons present in that nuclide.

Figure 13.7 shows the plot of proton number versus neutron number for the known nuclides. The black circles represent the stable nuclides while the white circles represent the unstable nuclides.

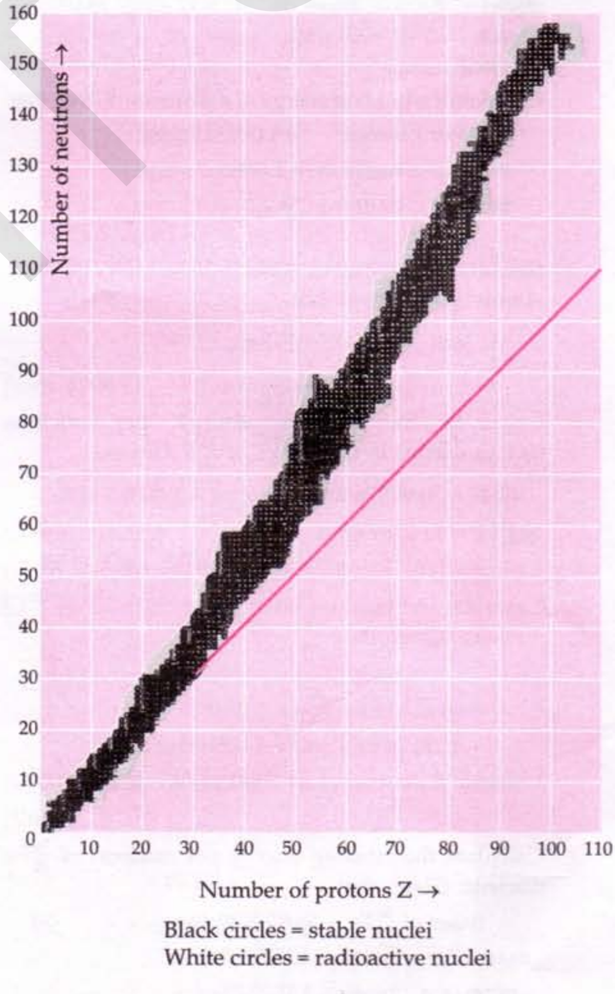

Fig. 13.7 Nuclidic chart.

The neutron-proton graph is called **nuclidic chart** *or* **Segre chart**. This graph reveals the following important features :

1. The stable nuclides lie on a well-defined narrow band and the unstable nuclides lie above and below this band.
2. The light stable nuclides tend to lie on the line $N = Z$. These nuclides have the same numbers of protons and neutrons so that the ratio $N/Z = 1$.
3. The ratio N/Z increases for heavier nuclides and becomes 1.6 for heaviest stable nuclides. In heavier nuclei, the proton number becomes large. The proton-proton repulsions are highly effective. So stability is achieved by having more neutrons than protons because neutrons do not undergo coulombic interactions.
4. The graph shows that there are no stable nuclei for $Z > 83$. Thus the heaviest stable nuclide is $^{209}_{83}\text{Bi}$.
5. The nuclides to the left of the stability region have excess neutrons, while those to the right of the stability region have excess protons. These nuclides are unstable and undergo radioactive disintegration. They are called *radioactive nuclides*.
6. The nuclides having even protons and even neutrons are most stable. About 60% of the known nuclides belong to this category.
7. Only four stable nuclides have both odd Z and odd N :

   $$^{2}_{1}\text{H},\ ^{6}_{3}\text{Li},\ ^{10}_{5}\text{Bi},\ ^{14}_{7}\text{N}$$

   These are called *odd-odd nuclides*. Also, there is no stable nuclide with $A = 5$ and $A = 8$.

For Your Knowledge

- To overcome proton-proton coulombic repulsions, heavier nuclei tend to achieve stability by having more neutrons than protons. So the ratio N/Z increases with A for stable nuclides. But a nucleus with *too many* neutrons is unstable because not enough of them are paired with protons. This increases the energy and hence, decreases the stability.
- All known nuclei with atomic numbers ranging from 1 to 117 have isotopes. Some of these have no stable isotopes. Stable isotopes occur for all atomic numbers between $Z = 1$ (hydrogen) and $Z = 83$ (bismuth) with the exception of $Z = 43$ and 61. All the isotopes with $Z = 84$ to 117 are radioactive. The total number of known isotopes is over 2500. Most of these are radioactive and only 266 isotopes are stable.

13.14 RADIOACTIVITY

16. *Define the term radioactivity. Briefly describe the circumstances which led to the discovery of this phenomenon.*

Radioactivity. *Radioactivity is the phenomenon of spontaneous disintegration of the nucleus of an atom with the emission of one or more penetrating radiations like α-particles, β-particles or γ-rays.*

A naturally occurring heavy nucleus is unstable. It spontaneously emits a particle, without the stimulus of any outside agency, transforming itself into a different nucleus. Such a nucleus is said to be ***radioactive*** and the process of transformation is called ***radioactive decay***. The process is spontaneous in the sense that it occurs by itself. It cannot be initiated, stopped, accelerated or retarded by changing

(*a*) the chemical conditions, or

(*b*) the physical conditions like temperature, pressure, etc ; other than the nuclear bombardment.

Discovery. The phenomenon of radioactivity was discovered accidentally by the French physicist *Henry Becquerel* in 1896. One day he left some pieces of uranium potassium sulphate wrapped in black paper in a drawer and separated the package from a photographic plate by a piece of silver. When he developed the photographic plate after several hours of exposure, he found, to his surprise, that the plate showed blackening due to some invisible radiations that must have been emitted by the uranium compound and were able to penetrate both the black paper and silver. These radiations were called ***Becquerel rays***. A couple of years later, the husband-and-wife team of *Pierre* and *Marie Curie* painstakingly isolated two new elements, radium and polonium. Of these, radium was found to be million times more active than uranium. *The substances which spontaneously emit penetrating radiations were called* ***radioactive substances***, by the Curie couple. *The phenomenon of spontaneous emission of radiation by radioactive substances came to be known as* ***radioactivity***. The Nobel prize for 1903 was shared among the *three – Becquerel, Marie Curie* and *Pierre Curie*, for their work on Radioactivity.

Examples of radioactive substances are : uranium, polonium, radium, thorium, actinium, etc. It is seen that all naturally occurring elements with atomic number greater than 82 show radioactivity.

13.15 ELECTRICAL NATURE OF BECQUEREL RADIATIONS

17. *How can we establish experimentally that the radiation from a radioactive source consists of three distinct components ?*

Electrical nature of the radioactive radiation. *Rutherford* and *Villiard* were the first to analyse the radiation emitted by radium. This radiation was found to consist of three components :

1. A component which could hardly pass through 0.1 cm thick aluminium foil, called α-rays.
2. A component which was stopped by 5 mm thick aluminium sheet, called β-rays.
3. A component which could pass through even 30 cm thickness of an iron piece, called γ-rays.

A simple experimental arrangement to demonstrate the analysis of Becquerel radiation into three components is shown in Fig. 13.8.

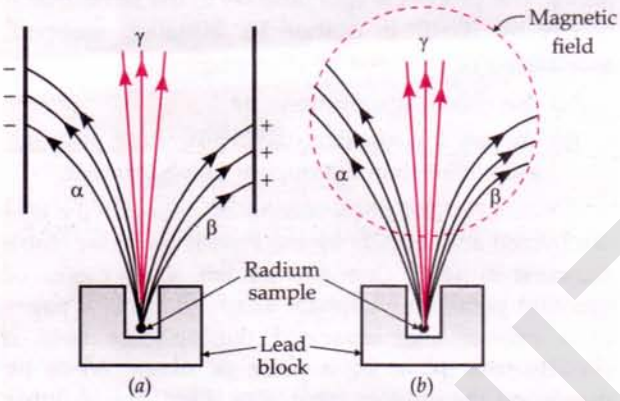

Fig. 13.8 Bending of Becquerel rays in (*a*) an electric field and (*b*) a magnetic field.

A small hole is drilled in lead block and a piece of radium is placed at its bottom. As the rays entering the walls of the lead block are absorbed before reaching the surface, only a narrow beam of radiation emerges from the hole. The beam is subjected to electric field [Fig. 13.8(*a*)] or magnetic field [Fig. 13.8(*b*)].

In both cases, the narrow beam splits into *three* components :

(*i*) The component which bends towards the left consists of positively charged particles, called α-rays.

(*ii*) The component which bends towards right consists of negatively charged particles, called β-rays.

(*iii*) The component which goes straight consists of neutral photons, called γ-rays.

13.16 PROPERTIES OF α-, β- AND γ-RAYS

18. *Mention the important properties of α-, β- and γ-rays.*

Properties of α-rays :

1. These are *positively charged particles.* These particles have been identified as *helium nuclei, i.e.,* doubly ionised helium atoms.
2. They are *deflected* by electric and magnetic fields. The directions of deflection indicate that α-particles are positively charged particles.
3. Their velocity is of the order of $\frac{1}{10}$th of the velocity of light.
4. They *excite fluorescence* in substances like zinc sulphide and barium platinocyanide.
5. They can *affect a photographic plate.*
6. They *ionise heavily* the gases through which they pass. An α-particle produces about 10^5 pairs of ions per cm of its path.
7. They *are easily absorbed by matter.* They are stopped by an aluminium foil of thickness 0.1 cm or by an ordinary sheet of paper.
8. They are *scattered* while passing through thin metal sheets.
9. They can cause *artificial disintegration* of an atom.
10. They produce *heating effect* when stopped and cause fatal burns on human body.
11. The *range* of α-particles in air, *i.e.,* the distance travelled by α-particles through air at S.T.P. before they lose their ionising power, varies from 2.70 cm (for uranium source) to 8.62 cm (for thorium source).

Properties of β-rays :

1. They consist of *fast moving electrons of nuclear origin.*
2. They are deflected by electric and magnetic fields. The direction of deflection indicates that β-rays are *negatively charged particles.*
3. They are emitted with a *range of velocities.* The maximum velocity depends on the nature of the radioactive source and may be as high as 99% of the speed of light.
4. They *excite fluorescence* in barium platinocyanide, calcium tungstate, etc.
5. They *effect a photographic plate* more strongly than α-particles.
6. They can *ionise a gas* but their ionising power is 1/100 times that of α-*rays.*
7. The penetrating power of β-particles is 100 times that of α-particles. They are absorbed by aluminium foil of 5 mm thickness.
8. The range of β-particles in air is much more than that of α-particles.
9. Due to their small masses, β-particles are easily scattered by atomic nuclei when passed through matter.
10. The emission of a β-particle is always accompanied by the emission of an elementary particle called *neutrino.*

Properties of γ -rays :

1. They are *electromagnetic waves* which have wavelength even less than that of X-rays.
2. They are *not deflected* by electric and magnetic fields, indicating that γ-particles (photons) do not carry any charge.
3. They travel with the speed of light.
4. They *excite fluorescence* in certain substances.
5. They *affect a photographic plate* even more strongly than β-rays.
6. They *ionise* gases very slightly. Their ionising power is $\frac{1}{10,000}$ times that of α-rays.
7. Their penetrating power is about 10,000 times that of α-rays. They can penetrate a 30 cm thick iron block.
8. Like X-rays, they are *diffracted by crystals.*
9. They *eject β -particles* from substances on which they fall.
10. They show the phenomenon of pair production. When a γ-ray photon passes close to a nucleus, it gets transformed into an elementary particle and its antiparticle.

    For example,

$$\underset{\gamma-\text{photon}}{\gamma} \longrightarrow \underset{\text{electron}}{{}_0e^-} + \underset{\text{positron}}{{}_0e^+}$$

13.17 COMPARISON BETWEEN THE PROPERTIES α-, β - AND γ-RAYS

19. *Compare and contrast the nature of α-, β- and γ-radiations.*

Property	α-Rays	β- Rays	γ-Rays
1. *Nature*	Helium nuclei	Electrons of nuclear origin	High energy e.m. radiations
2. *Mass*	6.67×10^{-27} kg or 4 amu	9.11×10^{-31} kg	Rest mass is zero
3. *Charge*	+ 2*e*	−*e*	0
4. *Deflection by $\vec{E}$ and $\vec{B}$*	Deflected towards –ve pole	Deflected towards +ve pole	Nil
5. *Speed*	$\approx 10^7$ ms^{-1}	$\approx 10^8$ ms^{-1} but variable	3×10^8 ms^{-1}
6. *Ionising power*	10^4 times that of γ-rays	10^2 times that of γ-rays	Minimum
7. *Penetrating power*	Minimum	10^2 times that of α-rays	10^4 times that of γ-rays
8. *Effect on photographic plate and ZnS phosphor*	Strong effect	Less effect	Least effect

13.18 SODDY-FAJAN'S DISPLACEMENT LAWS

20. *State Soddy-Fajan's displacement laws for radioactive transformations.*

Soddy-Fajan's displacement laws. According to *Rutherford-Soddy* theory whenever a radioactive disintegration occurs, it does so with the emission of an α- or a β-particle. The original nucleus is called *parent* and the new nucleus formed after disintegration is called *daughter*. Rutherford and Soddy used the following two rules to infer the nature of daughter nucleus from the parent nucleus and the particle emitted :

1. The algebraic sum of the charges before the disintegration must equal the total electric charge after the disintegration.
2. The sum of the mass numbers of the initial particles must equal the sum of the mass numbers of the final particles.

On the basis of these rules, *Soddy* and *Fajan* in 1913, gave simple **displacement laws for radioactive transformations**, which can be stated as follows :

1. *When a radioactive nucleus emits an α-particle, its atomic number decreases by 2 and mass number decreases by 4.*
2. *When a radioactive nucleus emits a β-particle, its atomic number increases by 1 but mass number remains the same.*
3. *The emission of a γ-particle does not change the mass number or the atomic number of the radioactive nucleus.*

In addition, the following points about radioactive disintegration may also be noted :

1. No *individual atom* can simultaneously emit both α- and β-particles.
2. *Different atoms* of the same element can emit either an α-particle or a β-particle.
3. This emission of a β-particle is usually accompanied by the emission of a γ-ray photon.

Illustrative example. A radioactive nucleus undergoes a series of decays according to the scheme :

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

If the mass number and atomic number of A are 180 and 72 respectively, what are these numbers for A_4 ?

Sol. The decay processes may be represented as

$$^{180}_{72}A \xrightarrow[\alpha]{^4_2\text{He}} {}^{176}_{70}A_1 \xrightarrow[\beta^-]{^{0}_{-1}e} {}^{176}_{71}A_2$$

$$\xrightarrow[\alpha]{^4_2\text{He}} {}^{172}_{69}A_3 \xrightarrow[\gamma]{} {}^{172}_{69}A_4$$

∴ Mass number of A_4 = **172**

Atomic number of A_4 = **69.**

13.19 RADIOACTIVE DECAY LAW

21. *State and deduce radioactive decay law. Hence define disintegration constant.*

Radioactive decay law. According to *Rutherford-Soddy theory* (*i*) The radioactive atoms are unstable and they decay spontaneously to emit α- or β-particles alongwith γ-rays. (*ii*) The disintegration is random. It is purely a matter of chance for any atom to disintegrate first. (*iii*) The disintegration is independent of all physical and chemical conditions and so it can neither be accelerated nor retarded.

The above facts show that it is not possible to predict whether a particular nucleus will decay in a given time interval. By using the concept of probability, the decay behaviour of a collection of a large number of nuclei can be predicted accurately in terms of the ***radioactive decay law*** which states :

The number of nuclei disintegrating per second of a radioactive sample at any instant is directly proportional to the number of undecayed nuclei present in the sample at that instant.

Let

N_0 = the number of radioactive nuclei present initially at time $t = 0$ in a sample of radioactive substance.

N = the number of radioactive nuclei present in the sample at any instant t, and

dN = the number of radioactive nuclei which disintegrate in the small time interval dt.

According to radioactive law, the rate of decay at any instant is proportional to the number of undecayed nuclei, *i.e.,*

$$-\frac{dN}{dt} \propto N$$

or $$-\frac{dN}{dt} = \lambda N \quad ...(1)$$

where λ is a proportionality constant called the *decay* or *disintegration constant.* Here the negative sign shows that the number of undecayed nuclei, N decreases with time. The equation (1) can be written as

$$\frac{dN}{N} = -\lambda\, dt$$

Integrating, $$\int \frac{dN}{N} = -\lambda \int dt$$

or $$\log_e N = -\lambda t + C \quad ...(2)$$

where C is a constant of integration.

At $t = 0$, $N = N_0$, therefore from equation (2), we get

$$\log_e N_0 = C$$

Then the equation (2) becomes

$$\log_e N = -\lambda t + \log_e N_0$$

or $$\log_e \frac{N}{N_0} = -\lambda t$$

or $$\frac{N}{N_0} = e^{-\lambda t}$$

or $$N = N_0 e^{-\lambda t} \quad ...(3)$$

This equation represents the *radioactive decay law.* It gives the number of active nuclei left after time t.

Fig. 13.9 shows a graph between the number N of undecayed nuclei and time t. It reveals the following features :

1. The number of active nuclei in a radioactive sample decreases exponentially with time. The disintegration is fast in the beginning but becomes slower and slower with the passage of time.
2. The larger the value of decay constant λ, the higher is the rate of disintegration.
3. Irrespective of its nature, a radioactive sample will take infinitely long time to disintegrate completely.

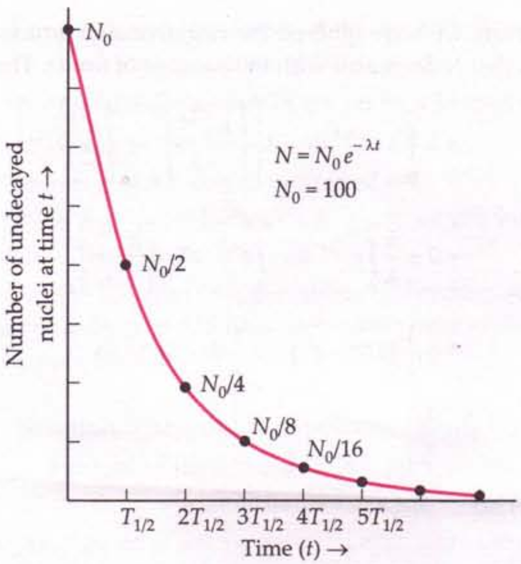

Fig. 13.9 Decay curve for a radioactive element.

Decay or disintegration constant :

If in equation (3), $t = \frac{1}{\lambda}$, then

$$N = N_0 e^{-1} = \frac{N_0}{e} = \frac{N_0}{2.718} = 0.368\ N_0$$

or $$N = \frac{N_0}{e} = 36.8\% \text{ of } N_0 \qquad ...(4)$$

*The **radioactive decay constant** may be defined as the reciprocal of the time interval during which the number of active nuclei in a given radioactive sample reduces to 36.8% (or 1/e times) of its initial value.*

Decay constant may be defined in another way also, as follows :

As $$-\frac{dN}{dt} = \lambda N$$

$$\therefore \qquad \lambda = \frac{-\frac{dN}{dt}}{N} \qquad ...(5)$$

Thus *the radioactive decay constant may be defined as the ratio of the instantaneous rate of disintegration to the number of active nuclei present in the radioactive sample at the given instant.* It gives the *probability per unit time* for a nucleus of a radioactive substance to decay. The value of λ depends on the nature of the radioactive substance.

13.20 HALF-LIFE

22. *Define half-life of a radioactive substance. Deduce its relation with disintegration constant. What is the significance of half-life ?*

Half-life. *The time interval in which one-half of the radioactive nuclei originally present in radioactive sample disintegrate is called half-life of the radioactive substance.* The half-life of a particular radioactive isotope is a characteristic constant of that isotope. It is denoted by $T_{1/2}$.

Relation between half-life and decay constant. Let

N_0 = Number of radioactive nuclei present in the radioactive sample initially (at $t = 0$).

N = Number of radioactive nuclei left at any instant t.

At $$t = T_{1/2}, \quad N = \frac{N_0}{2}$$

Now $N = N_0 e^{-\lambda t}$, where λ is the radioactive decay constant.

$$\therefore \quad \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \quad \text{or} \quad \frac{1}{2} = e^{-\lambda T_{1/2}}$$

or $$e^{\lambda T_{1/2}} = 2$$

Taking natural logarithm, we get

$$\lambda T_{1/2} \log_e e = \log_e 2$$

$$T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{2.303 \log 2}{\lambda}$$

$$= \frac{2.303 \times 0.3010}{\lambda} \qquad [\because \log_e e = 1]$$

or $$T_{1/2} = \frac{0.693}{\lambda}.$$

Thus the half-life of a radioactive substance is inversely proportional to its decay constant and is independent of the number N_0, the number of radioactive nuclei present initially in the sample.

Significance of half-life. (*i*) The *value of the half-life of a radio isotope gives an idea of the relative stability of that isotope.* An isotope having longer half-life is more stable than the isotope with shorter half-life.

The half-life can be as long as 10^{10} years, which is the estimated age of the universe, and can be shorter than 10^{-15} s. For example,

$$T_{1/2}(\text{U}-238) = 4.5 \times 10^9 \text{ years}$$

$$T_{1/2}(\text{Ra}-226) = 1620 \text{ years}$$

$$T_{1/2}(\text{Rn}-232) = 3.8 \text{ days}$$

$$T_{1/2}(\text{Po}-212) = 3 \times 10^{-7} \text{s}$$

The radioactive elements whose half-life is short are not found in observable quantities in nature today. However, they have been seen in nature. *Tritium* and *plutonium* belong to this category.

(*ii*) After one half-life, the number of undecayed nuclei in a given radioactive sample reduces to $N_0/2$, in two half-lives it becomes $N_0/4$, in three half-lives, it becomes $N_0/8$, and so on.

$\therefore$ Number of radioactive nuclei left undecayed after n half-lives

$$= N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$$

where $t = n \times T_{1/2}$ = total time of n half-lives.

13.21 MEAN LIFE

23. *Define mean life of a radioactive example. Deduce its relation with decay constant and half-life.*

Mean life. All the nuclei of a radioactive sample do not disintegrate at the same time. While one nucleus may disintegrate right at the beginning and some other may disintegrate at the end of the process. So the life time of the different nuclei may vary from zero to infinity.

The average time for which the nuclei of a radioactive sample exist is called ***mean life*** *or* ***average life*** *of that sample. It is equal to the ratio of the combined age of all the nuclei to the total number of nuclei present in the given sample.* It is denoted by τ.

$$\text{Mean life} = \frac{\text{Sum of the lives of all the nuclei}}{\text{Total number of nuclei}}$$

Relation between mean life and decay constant. Suppose a radioactive sample contains N_0 nuclei at time $t = 0$. After time t, this number reduces to N. Furthermore, suppose dN nuclei disintegrate in time t to $t + dt$. As dt is small, so the life of each of the dN nuclei can be approximately taken equal to t.

$\therefore$ Total life of dN nuclei $= t\,dN$

$$\text{Total life of all the } N_0 \text{ nuclei} = \int_0^{N_0} t\,dN$$

$$\text{Mean life} = \frac{\text{Total life of all the } N_0 \text{ nuclei}}{N_0}$$

or $$\tau = \frac{1}{N_0}\int_0^{N_0} t\,dN$$

As $$N = N_0 e^{-\lambda t}$$

$\therefore$ $$dN = -\lambda N_0 e^{-\lambda t}\,dt$$

When $N = N_0$, $t = 0$ and when $N = 0$, $t = \infty$.

Changing the limits of integration in terms of time, we get

$$\tau = \frac{1}{N_0}\int_0^{\infty} t\lambda N_0 e^{-\lambda t}\,dt$$

Here we have ignored the negative sign which just tells that N decreases with the passage of time t. Thus

$$\tau = \lambda\int_0^{\infty} t e^{-\lambda t}\,dt = \lambda\left[\left\{\frac{te^{-\lambda t}}{-\lambda}\right\}_0^{\infty} - \int_0^{\infty}\frac{e^{-\lambda t}}{-\lambda}\,dt\right]$$

$$= 0 + \frac{\lambda}{\lambda}\int_0^{\infty} e^{-\lambda t}\,dt = \int_0^{\infty} e^{-\lambda t}\,dt = \left[\frac{e^{-\lambda t}}{-\lambda}\right]_0^{\infty}$$

$$= -\frac{1}{\lambda}[e^{-\infty} - e^0] = -\frac{1}{\lambda}[0 - 1]$$

or $$\tau = \frac{1}{\lambda}$$

Also $$T_{1/2} = \frac{0.693}{\lambda} = 0.693\,\tau$$

or $$\tau = \frac{T_{1/2}}{0.693} = 1.44\,T_{1/2}.$$

13.22 ACTIVITY OF A RADIOACTIVE SUBSTANCE

24. *Define decay rate or activity of a radioactive sample. Show that $R = N\lambda$. Name and define the various units of radioactivity.*

Decay rate or activity of a radioactive sample. *The rate of decay or activity of a sample is defined as the number of radioactive disintegrations taking place per second in the sample.*

If a radioactive sample contains N radioactive nuclei at any time t, then its decay rate or activity R at the same time t will be

$$R = -\frac{dN}{dt}$$

The negative sign shows that the activity of the sample decreases with the passage of time.

According to the radioactive decay law,

$$-\frac{dN}{dt} = \lambda N$$

$\therefore$ $$R = \lambda N$$

As $N = N_0 e^{-\lambda t}$, so we can write

$$R = \lambda N_0 e^{-\lambda t}$$

or $$R = R_0 e^{-\lambda t}$$

This is *another form of the radioactive decay law*. Here $R_0 = \lambda N_0$, is the decay rate at time $t = 0$ and R is the decay rate at any subsequent time t. Like N, obviously R also decreases exponentially with time.

Units of radioactivity. The various units of rate of decay or activity of a radioactive substance are as follows :

1. Becquerel (Bq). The SI unit for activity is ***becquerel***, named after the discoverer of radioactivity, *Henry Becquerel. One becquerel is defined as the decay rate of one disintegration per second.*

1 becquerel = 1 Bq = 1 decay per second

2. Curie (Ci). It is an older practical unit for activity named in honour of *Madame Marie Sklodowska Curie* (1867-1934). *One curie is the decay rate of* 3.7×10^{10} *disintegrations per second.*

$$1 \text{ curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ decays per second}$$
$$= 3.7 \times 10^{10} \text{ Bq}$$

Some other units of activity in common use are

$$1 \text{ m Ci (milli curie)} = 3.7 \times 10^7 \text{ Bq}$$
$$1 \,\mu \text{ Ci (micro curie)} = 3.7 \times 10^4 \text{ Bq}$$

3. Rutherford (rd). *One rutherford is the decay rate of* 10^6 *disintegrations per second.*

$$1 \text{ rd (rutherford)} = 10^6 \text{ decays per second} = 10^6 \text{ Bq}$$
$$1 \text{ Ci} = 3.7 \times 10^4 \text{ rd}$$

13.23 ALPHA DECAY

25. *Explain the process of α-decay. Give suitable examples.*

Alpha decay. *Alpha decay is a process in which an unstable nucleus transforms itself into a new nucleus by emitting an alpha particle* (a helium nucleus, ${}^4_2\text{He}$).

Since an α-particle has two protons and two neutrons, so after an α-decay, the parent nucleus is transformed into a daughter nucleus with mass number smaller by 4 and atomic number smaller by 2.

An alpha decay can be expressed by the equation :

$${}^A_Z X \longrightarrow {}^{A-4}_{Z-2}Y + {}^4_2\text{He} + Q$$

Here Q is the energy released in the process and can be determined from Einstein's mass-energy relation which gives

$$Q = [m_X - m_Y - m_{\text{He}}]c^2$$

where m_X, m_Y and m_{He} are the masses of the parent nucleus X, daughter nucleus Y and the α-particle respectively. The energy Q is shared by the daughter nucleus Y and the α-particle. As the parent nucleus is at rest before its α-decay, the α-particles are emitted with fixed energy. This energy can be determined by applying the laws of conservation of energy and momentum.

For example, uranium-238 on emitting an α-particle changes into thorium-234.

$${}^{238}_{92}\text{U} \longrightarrow {}^{234}_{90}\text{Th} + {}^4_2\text{He} + Q$$

Similarly, polonium – 208 is transmuted into lead – 204.

$${}^{208}_{84}\text{Po} \longrightarrow {}^{204}_{82}\text{Pb} + {}^4_2\text{He} + Q$$

Generally, the nuclei with mass number 210 or more undergo α-decay. In such nuclei, the long range repulsive forces between the protons dominate over the short range nuclear forces which bind the various nucleons together. By emitting α-particles, these nuclei achieve greater stability. An α-particle has a high value of binding energy (≈ 28 MeV). After the emission of an α-particle, the binding energy per nucleon increases and the residual nucleus becomes more stable.

26. *How can we estimate the speed of α-particles emitted in α-decay ?*

Speed of emitted α-particles. Consider the alpha decay :

$${}^A_Z X \longrightarrow {}^{A-4}_{Z-2}Y + {}^4_2\text{He} + Q$$

The speed of the emitted α-particles can be calculated by using the laws of conservation of energy and momentum. Suppose the parent nucleus ${}^A_Z X$ be at rest before decay. Let v_{He} and v_Y be the velocities of the α-particle and the daughter nucleus. Applying the law of conservation of momentum, we get

$$m_Y v_Y = m_{He} v_{He} \qquad ...(1)$$

As the energy Q released in the decay process appears in the form of kinetic energy of α-particle and the daughter nucleus, so we have

$$\frac{1}{2} m_{He} v_{He}^2 + \frac{1}{2} m_Y v_Y^2 = Q$$

Substituting the value of v_Y from Eq. (1), we get

$$\frac{1}{2} m_{He} v_{He}^2 + \frac{1}{2} m_Y \frac{m_{He}^2 v_{He}^2}{m_Y^2} = Q$$

or $$\frac{1}{2} m_{He} m_Y v_{He}^2 + \frac{1}{2} m_{He}^2 v_{He}^2 = m_Y Q$$

or $$\frac{1}{2}(m_Y + m_{He}) m_{He} v_{He}^2 = m_Y Q$$

or $$K_{He} = \frac{1}{2} m_{He} v_{He}^2 = \frac{m_Y}{m_Y + m_{He}} \cdot Q$$

Now $m_Y \simeq (A-4)$ amu and $m_{He} \simeq 4$ amu, therefore,

$$K_{He} = \frac{1}{2} m_{He} v_{He}^2 \simeq \frac{(A-4)}{A} \cdot Q$$

$$\therefore \quad v_{He} = \sqrt{\frac{2 K_{He}}{m_{He}}} = \sqrt{\frac{2(A-4)Q}{A\, m_{He}}}$$

For example, in the α-decay of a radon nucleus ${}^{222}_{86}\text{Rn}$, we have

$$Q = 5.587 \text{ MeV}$$

$$\therefore \quad K_{He} = \frac{A-4}{A} Q = \frac{(222-4)}{222} \times 5.587 \text{ MeV}$$

$$= 5.486 \text{ MeV} = 5.486 \times 1.6 \times 10^{-19} \text{ J}$$

$$m_{He} = 4 \text{ amu} = 4 \times 1.66 \times 10^{-27} \text{kg}$$

$$\text{Hence} \quad v_{He} = \sqrt{\frac{2 \times 5.486 \times 1.6 \times 10^{-19}}{4 \times 1.66 \times 10^{-27}}} \text{ ms}^{-1}$$

$$= \mathbf{1.62 \times 10^7 \text{ ms}^{-1}}.$$

27. *Explain qualitatively the emission of an α-particle from a radioactive nucleus on the basis of wave mechanics.*

Theory of α-decay : Tunnelling effect. The α-particles emitted by different radioactive nuclei have kinetic energy ranging from 4 to 9 MeV. The nucleus of an α-emitter poses a barrier of height about 25 MeV. Fig. 13.10 shows a plot of the potential energy U of the system consisting of the α-particle and the residual nucleus. The α-particles are short of about 16 to 25 MeV of energy, needed for the emission. Therefore, classically, we cannot explain the emission of α-particles by radioactive nuclei.

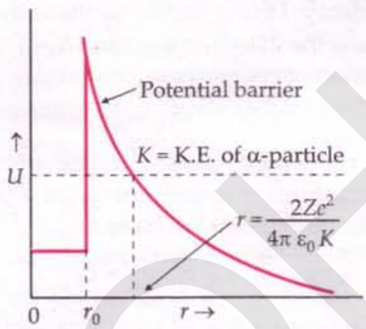

Fig. 13.10 Plot of potential energy U of an α-particle as a function of distance r from the centre of the residual nucleus.

In 1928, *Gamow, Congdon* and *Gurney* explained the emission of α-particles in terms of the penetration of the nuclear potential barrier on the basis of quantum theory. According to this theory :

1. An α-particle may exist as an entity (already formed) inside a nucleus before it escapes from the nucleus.
2. The α-particle is in a state of constant motion inside the nucleus with a speed of about 10^7ms^{-1}.
3. Quantum mechanically, an α-particle of even insufficient kinetic energy has a small but finite probability p of its crossing the potential barrier.

As the size of the nucleus $\approx 10^{-14}$m and speed of α-particle $\approx 10^7 \text{ms}^{-1}$, the α-particle takes about 10^{-21}s to move across the nucleus. Thus α-particle presents itself before the potential barrier 10^{21} times in a second. The probability of escape of an α-particle from a nucleus will be

$$P = p \times \nu$$

As ν is large (10^{21}s^{-1}), so P is sufficiently large and the α-particle can tunnel through the energy barrier which is classically insurmountable. *Hence α-decay occurs as a result of barrier tunnelling.*

The barrier tunnelling explains why every $^{238}_{92}U$ nuclide in a sample of $^{238}_{92}U$ atoms does not decay at once, even when its decay process has a positive Q value. Consequently, the half-lives for α-decay of most of the alpha unstable nuclei are very long. For example, the half-life of $^{238}_{92}U$ for α-decay is 4.5×10^9 years.

13.24 BETA DECAY

28. *Explain β-decay. How can radioactive nuclei emit β-particles even though nuclei do not contain these particles ? Hence explain why the mass number of a radioactive nuclide does not change during β-decay. Why is the energy distribution of β-rays continuous ?*

Beta decay. *The process of spontaneous emission of an electron (e^-) or a positron (e^+) from a nucleus is called β-decay.*

Like α-decay, β-decay is a spontaneous process, with a definite disintegration energy and half-life. It is also a statistical process, obeying the law of radioactive decay.

In ***beta minus*** (β^-) ***decay***, the mass number of the radioactive nucleus remains unchanged but its atomic number increases by one. An electron and a new particle antineutrino ($\bar{\nu}$) are emitted from the nucleus, as in the decay :

$$^{32}_{15}P \longrightarrow {}^{32}_{16}S + e^- + \bar{\nu} \qquad [T_{1/2} = 2.6 \text{ days}]$$

In general, the beta minus decay may be represented as

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \bar{\nu}$$

$$\text{or} \qquad ^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + \beta^- + \bar{\nu}$$

The electron emitted from the nucleus is called a ***beta minus particle***, denoted by β^-.

In ***beta plus*** (β^+) ***decay***, the mass number of the radioactive nucleus remains unchanged but its atomic number decreases by one. A positron (e^+) and a new particle neutrino (ν) are emitted from the nucleus, as in the decay :

$$^{22}_{11}Na \longrightarrow {}^{22}_{10}Ne + e^+ + \nu \qquad [T_{1/2} = 2.6 \text{ years}]$$

In general, the beta plus decay may be represented as

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}e + \nu$$

or $$^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + \beta^{+} + \nu$$

The positron so emitted is called a ***beta plus particle*** (β^+). The positron is an antiparticle of electron. It has a positive charge equal in magnitude to the charge on an electron and has a mass equal to the mass of an electron. Similarly, neutrino and antineutrino are antiparticles of each other. Both are massless, chargeless particles having spins $\pm\frac{1}{2}$.

Although a nucleus contains no electrons, positrons and neutrinos, yet can it eject these particles. It is believed that electrons, positrons and neutrinos are created during the process of beta decay. If the unstable nucleus has excess neutrons than needed for stability, a neutron converts itself into a proton. So in a ***beta-minus decay***, an *electron* and an *antineutrino* are created and emitted from the nucleus via the reaction :

$$n \longrightarrow p + e^{-} + \bar{\nu}$$

If the unstable nucleus has excess protons than that needed for stability, a proton converts itself into a neutron. So in a ***beta-plus decay***, a *positron* and a *neutrino* are created and emitted from the nucleus via the reaction :

$$p \longrightarrow n + e^{+} + \nu$$

Clearly, a beta decay process involves the conversion of a neutron into a proton or vice versa. These nucleons have nearly equal masses. That is why *the mass number A of a nuclide undergoing beta decay does not change.*

Continuous energy spectrum for beta rays. In both α- and β-decays, the disintegration energy Q depends on the nature of the radionuclide. In the α-decay of a particular radionuclide, every emitted α-particle has a definite amount of kinetic energy. However in β-decay, the disintegration energy is shared in all proportions between the three particles : daughter nucleus, electron (or positron) and antineutrino (or neutrino). As a result, the kinetic energy of the electrons (or positrons) is not fixed. Their energy varies from zero to a maximum value K_{max}. Thus β-rays have a *continuous energy spectrum*, as shown in Fig. 13.11. The maximum kinetic energy or end point energy K_{max} must be equal to disintegration energy Q. When the electron (or positron) has maximum energy, the energy carried by the daughter nucleus and neutrino is nearly zero.

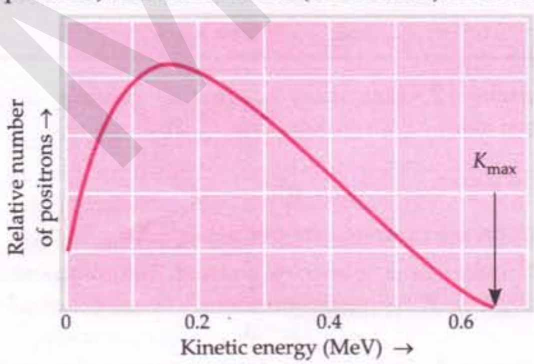

Fig. 13.11 The distribution of the kinetic energies of positrons emitted in the decay of $^{64}_{29}Cu$.

For Your Knowledge

- **Pauli's neutrino hypothesis.** In a given β-decay reaction, the energy of the electrons is expected to be fixed one. For example, consider the β-decay of $^{210}_{83}Bi$ into $^{210}_{84}Po$:

$$^{210}_{83}Bi \longrightarrow {}^{210}_{84}Po + {}^{0}_{-1}e$$

In the above reaction, the electrons are expected to come out with a fixed energy of 1.17 MeV because here electron is the only particle that comes out of the nucleus and it should carry whole of the disintegration energy. However, experiments showed that the energy of the emitted electrons varies from zero to a maximum value of 1.17 MeV.

Moreover, an electron has spin equal to 1/2, so to conserve angular momentum, the spin of radio nucleus must change by 1/2. But in actual practice, there is either no change or the spin changes by an integral value.

Thus the laws of conservation of energy and angular momentum were not found to be obeyed in β-decay. To remove this discrepancy, *Pauli* in 1930 suggested that an uncharged particle of zero rest mass and spin 1/2 is emitted alongwith the electron. This particle was named *antineutrino* ($\bar{\nu}$). The antineutrino can carry away different amounts of energy, leaving the electrons with different energies. Hence the energy distribution of β-rays is continuous.

- Since neutrinos (or antineutrinos) are massless and chargeless, they interact so weakly with matter that it becomes very difficult to detect them. They can penetrate through earth without being absorbed. By ingenious experiments, neutrinos have been detected and their mass and spin or intrinsic angular momentum have been measured.

- **Electron capture.** Some proton rich nuclei capture one of the atomic electrons (usually from the K shell). A proton in the nucleus combines with this electron forming a neutron. A neutrino created in the process is emitted from the nucleus.

$$p + e^{-} \rightarrow n + \nu$$

The entire process may be represented as

$$^{A}_{Z}X + e^{-} \rightarrow {}^{A}_{Z-1}Y + \nu$$

This process is called ***electron capture*** or ***K- capture***. The vacancy created in the K shell is filled by transition of electrons from the outer shells. This results in the emission of ***characteristic X-rays***.

13.25 GAMMA DECAY

29. *What is gamma decay ? Explain it with the help of an example.*

Gamma decay. *The process of emission of a γ-ray photon during the radioactive disintegration of a nucleus is called gamma decay.*

As the emitted γ-ray photons have zero rest mass and carry no charge, so in a γ-decay the mass number and atomic number of the nucleus remain unchanged and no new element is formed. A γ-decay can be expressed as

$$\underset{\text{(Excited state)}}{{}^{A}_{Z}X} \longrightarrow \underset{\text{(Ground state)}}{{}^{A}_{Z}X} + \gamma$$

A nucleus does not contain photons, yet it can emit photons. These photons are created during the emission process. We know that a nucleus can exist in different energy states. After an α- or a β-decay, the daughter nucleus is usually left in the *excited state*. It attains the *ground state* by single or successive transitions by *emitting one or more photons*. As the nuclear states have energies of the order of MeV, therefore, the photons emitted by the nuclei have energy of the order of several MeV. The wavelength of such high energy photons is a fraction of an angstrom. The short wavelength electromagnetic waves emitted by nuclei are called γ-rays.

An example of γ-decay is shown through an energy level diagram in Fig. 13.12. Here an unstable ${}^{60}_{27}Co$ nucleus is transformed via a β-decay into an excited ${}^{60}_{28}Ni$ nucleus, which in turn reaches the stable ground state by emitting photons of energies 1.17 MeV and 1.33 MeV, in two successive γ-decay processes.

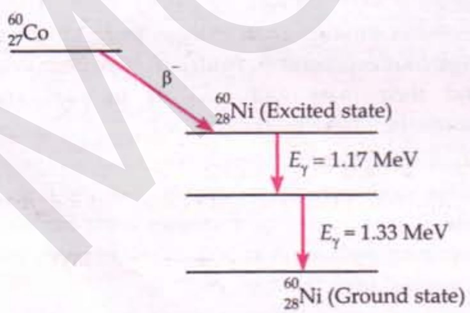

Fig. 13.12 Energy-level diagram showing the emission of γ-rays by a ${}^{60}_{27}Cu$ nucleus subsequent to beta decay.

Usually, γ-rays are emitted after α- or β-decay, but there are long lived radioactive nuclei that emit only γ-rays.

Examples based on Radioactivity

Formulae Used

1. Displacement laws for radioactive transformations are as follows :

   (*i*) α-decay : ${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$

   (*ii*) β-decay : ${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \bar{\nu}$

   (*iii*) γ-decay : $\underset{\text{(Excited state)}}{{}^{A}_{Z}X} \longrightarrow \underset{\text{(Ground state)}}{{}^{A}_{Z}X} + \gamma$

2. Radioactive decay law :

   (*i*) $-\frac{dN}{dt} = \lambda N$ (*ii*) $N = N_0 e^{-\lambda t}$

   where λ = decay constant or disintegration constant

3. Half life : $T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$

4. $N = N_0 \left(\frac{1}{2}\right)^n$, where $n = \frac{t}{T_{1/2}}$

5. Mean life : $\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44\, T_{1/2}$

   or $T_{1/2} = 0.693\,\tau$

6. Decay rate or activity of a substance :

   $$R = \left|\frac{dN}{dt}\right| = \lambda N = \lambda N_0 e^{-\lambda t}$$

7. Time required to reduce the radioactive substance,

   $$t = \frac{2.303}{\lambda} \log \frac{No}{N}$$

8. Decay constant : $\lambda = \frac{2.303}{t} \log \frac{No}{N}$.

Units Used

Times t, $T_{1/2}$ and τ are in second, decay constant λ in s^{-1}, decay rate in curie or rutherford.

1 Ci (curie) = 3.70×10^{10} disintegrations/s

1 rd (rutherford) = 10^6 disintegrations/s

Example 12. *How many α – and β⁻ –particles will be emitted when ${}^{232}_{90}Th$ changes into ${}^{208}_{82}Pb$?*

Solution. ${}^{232}_{90}Th \rightarrow {}^{208}_{82}Pb$

Decrease in mass number = 232 – 208 = 24

Number of α –particles emitted due to the above decrease in mass number $= \frac{24}{4} = 6$

Expected decrease in atomic number due to the emission of 6 α –particles = 6 × 2 = 12

Expected atomic number of the nucleus formed

$$= 90 - 12 = 78.$$

But the atomic number of the nucleus formed $= 82$

Increase in atomic number $= 82 - 78 = 4$

Number of β^- –particles emitted $= 4$

Thus 6 α –particles and 4 β^--particles are emitted when ${}^{232}_{90}\text{Th}$ changes into ${}^{208}_{82}\text{Pb}$.

Example 13. *Half-life of a certain radioactive material against α-decay is 138 days. After what lapse of time the undecayed fraction of the material will be 6.25% ?* [CBSE OD 90]

Solution. The half-life of polonium is 38 days.

∴ Amount of undecayed Po left after 138 days

$= 50\%$ of initial amount

Amount of undecayed Po left after next 138 days

$= 25\%$ of initial amount

Amount of undecayed Po left after next 138 days

$= 12.5\%$ of initial amount

Amount of undecayed Po left after next 138 days

$= 6.25\%$ of initial amount

∴ Total time lapsed $= 138 \times 4 =$ **552 days.**

Example 14. *The half-life of radium is 1600 years. After how many years 25% of a radium block remains undecayed ?*

Solution. Here $N = 25\%$ of $N_0 = \dfrac{N_0}{4}$

As $\quad N = N_0\left(\dfrac{1}{2}\right)^n \quad \therefore \quad \dfrac{N_0}{4} = N_0\left(\dfrac{1}{2}\right)^n$

or $\quad \left(\dfrac{1}{2}\right)^2 = \left(\dfrac{1}{2}\right)^n \quad \therefore \quad n = 2$

Time of disintegration

= Half-life × Number of half-lives

$= 1600 \times 2 =$ **3200 years.**

Example 15. *Find the half life of a radioactive material if its activity drops to 1/16th of its initial value in 30 years.*

Solution. As activity ∝ No. of atoms present.

$$\therefore \quad N = \frac{N_0}{16}$$

But $\quad N = N_0\left(\dfrac{1}{2}\right)^n,$

where n is the number of half lives

$$\therefore \quad \frac{N_0}{16} = N_0\left(\frac{1}{2}\right)^n \text{ or } \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n \text{ or } n = 4$$

Half-life period

$$= \frac{\text{Time of disintegration}}{\text{No. of half lives}} = \frac{30}{4} = \textbf{7.5 years}.$$

Example 16. *The half-life, of a given radioactive nuclide, is 138.6 days. What is the mean life of this nuclide ? After how much time will a given sample of this radioactive nuclide get reduced to only 12.5% of its initial value ?* [CBSE OD 04C]

Solution. Here $T_{1/2} = 138.6$ days

Mean life,

$$\tau = 1.44\, T_{1/2} = 1.44 \times 138.6 = \textbf{199.58 days}.$$

Given $\dfrac{N}{N_0} = 12.5\% = \dfrac{12.5}{100} = \dfrac{1}{8}$

But $\quad \dfrac{N}{N_0} = \left(\dfrac{1}{2}\right)^n$

$$\therefore \quad \frac{1}{8} = \left(\frac{1}{2}\right)^n$$

or $\quad \left(\dfrac{1}{2}\right)^3 = \left(\dfrac{1}{2}\right)^n$

∴ Number of half-lives, $n = 3$

Time taken = Half life × Number of half-lives

$= 138.6 \times 3 =$ **415.8 days.**

Example 17. *It is observed that only 6.25% of a given radioactive sample is left undecayed after a period of 16 days. What is the decay constant of this sample, in day^{-1} ?* [CBSE OD 07C]

Solution. Here $\dfrac{N}{N_0} = 6.25\%$

or $\quad \left(\dfrac{1}{2}\right)^n = \dfrac{6.25}{100} = \dfrac{1}{16} = \left(\dfrac{1}{2}\right)^4 \quad \therefore \quad n = 4$

$$T_{1/2} = \frac{t}{n} = \frac{16 \text{ days}}{4} = 4 \text{ days}$$

Decay constant,

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{4} = \textbf{0.173 day}^{-1}.$$

Example 18. *The decay constant, for a given radioactive sample, is 0.3465 day^{-1}. What percentage of this sample will get decayed in a period of 4 years ?* [CBSE OD 07C]

Solution. Here $\lambda = 0.3465$ day^{-1}, $t = 4$ years

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.3465} = 2 \text{ days}$$

$$\therefore \quad n = \frac{t}{T_{1/2}} = \frac{4}{2} = 2$$

Hence sample left undecayed after a period of 4 years,

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 25\%.$$

Example 19. *A radioactive material is reduced to 1/16 of its original amount in 4 days. How much material should one begin with so that 4×10^{-3} kg of the material is left after 6 days.* [CBSE Sample Paper 08]

Solution. As $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$ $\therefore$ $\frac{1}{16} = \left(\frac{1}{2}\right)^n$

or $\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n$ or $n = 4$

$$T_{1/2} = \frac{t}{n} = \frac{4 \text{ days}}{4} = 1 \text{ day}$$

Now $N = 4 \times 10^{-3}$ kg, $t = 6$ days or $n = 6$

$\therefore$ $N_0 = N\,(2)^n = 4 \times 10^{-3} \times 2^6 = \mathbf{0.256\ kg.}$

Example 20. *The half life of radioactive substance is 20 s. Calculate :*

(*i*) *the decay constant, and*

(*ii*) *time taken for the sample to decay by 7/8th of the initial value.* [CBSE F 09]

Solution. (*i*) Given, $T_{1/2} = 20$ s

Decay constant, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{20} = \mathbf{0.0346\ s^{-1}}$.

(*ii*) After three half lives, the fraction of undecayed nuclei

$$= \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$\therefore$ Time taken for the sample to decay by $\left(1 - \frac{1}{8}\right)$ or $\frac{7}{8}$th of the initial value

$$= 3T_{1/2} = 3 \times 20 = \mathbf{60\ s.}$$

Example 21. *An observer in a laboratory starts with N_0 nuclei of a radioactive sample and keeps on observing the number (N) of left over nuclei at regular intervals of 10 minutes each. She prepares the following table on the basis of her observations :*

Time (t) (in min.)	0	10	20	30	40
$\log_e\left(\frac{N_0}{N}\right)$	0	3.465	6.930	10.395	13.860

Use this data to plot a graph of $\log_e(N_0/N)$ vs. time (t) and calculate the (i) decay constant and (ii) half-life of the given sample. [CBSE D 09C]

Solution. The graph between $\log_e(N_0/N)$ and time (t) is shown in Fig. 13.13.

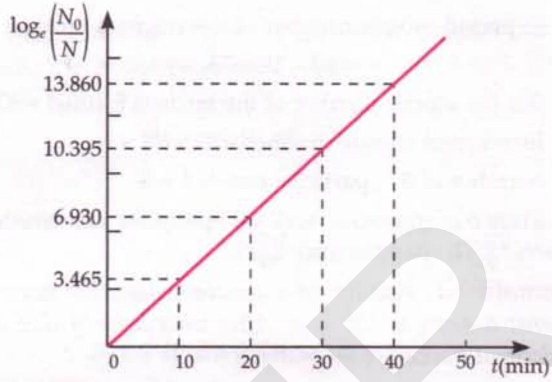

Fig. 13.13

(*i*) Now $\log_e \frac{N_0}{N} = \lambda t$

$\therefore$ $\lambda = \frac{1}{t} \log_e \frac{N_0}{N}$

When $t = 10$ min,

$$\log_e \frac{N_0}{N} = 3.465$$

$\therefore$ $\lambda = \frac{1}{10} \times 3.465 = \mathbf{0.3465\ min^{-1}}$.

(*ii*) $T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.3465} = \mathbf{2\ min.}$

Example 22. *The half-life of radium is 1500 years. After how many years will one gram of the pure radium*

(*i*) *reduce to one centigram ?*

(*ii*) *lose one milligram ?*

Solution. Half-life,

$$T_{1/2} = 1500 \text{ years}$$

Decay constant,

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1500} \text{ year}^{-1}$$

(*i*) Initial amount,

$$N_0 = 1 \text{ g}$$

Remaining amount, $N = 1$ centigram $= 0.01$ g

$\therefore$ Required time,

$$t = \frac{2.303}{\lambda} \log \frac{N_0}{N}$$

$$= \frac{2.303 \times 1500}{0.693} \log \frac{1}{0.01} \text{ years}$$

$$= \frac{2.303 \times 1500 \times 2}{0.693} \text{ years}$$

$$= \mathbf{9.972 \times 10^3\ years.}$$

(*ii*) Initial amount, $N_0 = 1$ g

Remaining amount, $N = 1 - 10^{-3} = 0.999$ g

$\therefore$ Required time,

$$t = \frac{2.303}{\lambda} \log \frac{N_0}{N} = \frac{2.303 \times 1500}{0.693} \log \frac{1}{0.999} \text{ years}$$

$$= \frac{2.303 \times 1500 \times 0.0004}{0.693} \text{ years } = \mathbf{1.995 \text{ years.}}$$

Example 23. *The decay constant, for a radionuclide, has a value of 1.386 day^{-1}. After how much time will a given sample of this radionuclide get reduced to only 6.25% of its present number ?* [CBSE OD 04C]

Solution. Here $\lambda = 1.386 \text{ day}^{-1}$

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{1.386 \text{ day}^{-1}} = 0.5 \text{ day}$$

$$\frac{N}{N_0} = 6.25\% = \frac{6.25}{100} = \frac{1}{16} = \left(\frac{1}{2}\right)^4$$

Required time,

$$t = \eta T_{1/2} = 4 \times 0.5 = \mathbf{2 \text{ days}}.$$

Example 24. *The half life of $^{238}_{92}U$ against α-decay is 1.5×10^{17} s. What is the activity of a sample of $^{238}_{92}U$ having 25×10^{20} atoms ?* [CBSE D 05]

Solution. Here $T_{1/2} = 1.5 \times 10^{17}$ s, $N = 25 \times 10^{20}$ atoms

$$\therefore \quad R = \lambda N = \frac{0.693}{T_{1/2}} \times N$$

$$= \frac{0.693 \times 25 \times 10^{20}}{1.5 \times 10^{17}}$$

$$= \mathbf{11550 \text{ disintegrations / second.}}$$

Example 25. *The half life of $^{238}_{92}U$ against α-decay is 4.5×10^9 years. Calculate the activity of 1 g sample of $^{238}_{92}U$.* [NCERT ; CBSE OD 05 ; F 06]

Solution. Here $T_{1/2} = 4.5 \times 10^9$ years

$$= 4.5 \times 10^9 \times 3.156 \times 10^7 \text{ s},$$

$$m = 1 \text{ g}, \; M = 238$$

Number of atoms in 1 g uranium,

$$N = \frac{m}{M} \times \text{Avogadro's number}$$

$$= \frac{1 \times 6.023 \times 10^{23}}{238} \text{ atoms}$$

Activity of the sample,

$$R = \lambda N = \frac{0.693}{T_{1/2}} . N$$

$$= \frac{0.693 \times 6.023 \times 10^{23}}{4.5 \times 3.156 \times 10^{16} \times 238} \text{ s}^{-1}$$

$$= \mathbf{1.235 \times 10^4 \text{ Bq.}}$$

Example 26. *A radioactive isotope has a half-life of 5 years. How long will it take the activity to reduce to 3.125% ?* [CBSE OD 08]

Solution. Since activity is proportional to the number of radioactive atoms, therefore

$$\frac{R}{R_0} = \frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

But $\quad \dfrac{R}{R_0} = 3.125\% = \dfrac{3.125}{100} = \dfrac{1}{32} = \left(\dfrac{1}{2}\right)^5$

$$\therefore \quad \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^5 \quad \text{or} \quad n = 5$$

Required time, $t = nT_{1/2} = 5 \times 5$ years $=$ **25 years.**

Example 27. *A radioactive sample contains 2.2 mg of pure $^{11}_{6}C$ which has half life period of 1224 seconds. Calculate :*

(i) *the number of atoms present initially.*

(ii) *the activity when 5 μg of the sample will be left.* [CBSE OD 05]

Solution. (i) Number of atoms present in 11 g of the sample $= 6.023 \times 10^{23}$

$\therefore$ Number of atoms present in 2.2 mg of the sample

$$= \frac{6.023 \times 10^{23} \times 2.2 \times 10^{-3}}{11}$$

$$= 1.2 \times 10^{20} \text{ atoms}$$

= Number of atoms present initially.

(ii) Number of atoms present in 5 μg of the sample

$$= \frac{6.023 \times 10^{23} \times 5 \times 10^{-6}}{11}$$

$$= 2.74 \times 10^{17} \text{ atoms}$$

Activity of the sample,

$$R = \lambda N = \frac{0.693}{T_{1/2}} \times N$$

$$= \frac{0.693 \times 2.74 \times 10^{17}}{1224}$$

$$= \mathbf{1.55 \times 10^{14} \text{ disintegrations / second.}}$$

Example 28. *In an experiment, the activity of 1.2 milligrams of radioactive potassium chloride (chloride of isotope K-40) was found to be 170 s^{-1}. Taking molar mass of to be 0.075 kg mole^{-1}, find the number of K-40 atoms in the same and hence find the half-life of K-40. Avogadros' number $= 6.0 \times 10^{23}$ mole^{-1}.* [ISCE 97]

Solution. Molar mass of K-40 Cl,

$$M = 0.075 \text{ kg mol}^{-1} = 75 \text{ g mol}^{-1}$$

Number of molecules present in 1.2 mg of potassium chloride

$$N = \frac{m}{M} \times \text{Avogadro's number}$$

$$= \frac{1.2 \times 10^{-3} \times 6.0 \times 10^{23}}{75} = 9.6 \times 10^{18}$$

Given $\quad R = 170\ s^{-1}$

But $\quad R = \lambda N = \frac{0.693}{T_{1/2}} . N$

$$\therefore \quad T_{1/2} = \frac{0.693\ N}{R} = \frac{0.693 \times 9.6 \times 10^{18}}{170}$$

$$= \mathbf{3.91 \times 10^{16}\ s.}$$

Example 29. *Some amount of a radioactive substance (half-life = 10 days) is spread inside a room and consequently the level of radiation becomes 50 times the permissible level for normal occupancy of the room. After how many days the room will be safe for occupation ?* [REC 92]

Solution. Let t be the time required to reach the permissible level. This means that the activity will drop to 1/50 of its present value after time t, *i.e.,*

$$\frac{R}{R_0} = \frac{1}{50}$$

But $R = N\lambda$ and $R_0 = N_0\lambda$.

Hence $\quad \frac{N}{N_0} = \frac{1}{50}$

As $\quad N = N_0 e^{-\lambda t}$

$$\therefore \quad \lambda t = \log_e \frac{N_0}{N} = \log_e 50$$

or $\quad t = \frac{\log_e 50}{\lambda}$

But $\quad \lambda = \frac{\log_e 2}{T_{1/2}}$

$$\therefore \quad t = T_{1/2} \frac{\log_e 50}{\log_e 2} = T_{1/2} \frac{\log_{10} 50}{\log_{10} 2}$$

$$= 10 \text{ days} \times \frac{1.6990}{0.3010} = \mathbf{56.45\ days.}$$

Example 30. *We are given the following atomic masses :*

$^{238}_{92}U = 238.05079$ *amu* $\qquad$ $^{4}_{2}He = 4.00260$ *amu*

$^{234}_{90}Th = 234.04363$ *amu* $\qquad$ $^{1}_{1}H = 1.00783$ *amu*

$^{237}_{91}Pa = 237.03121$ *amu*

Here the symbol Pa is for the element protactinium (Z = 91).

(*a*) *Calculate the energy released during the α-decay of* $^{238}_{92}U$. [CBSE OD 08]

(*b*) *Calculate the kinetic energy of the emitted α-particles.*

(*c*) *Show that* $^{238}_{92}U$ *cannot spontaneously emit a proton.* [NCERT]

Solution. (*a*) The equation for the α-decay of $^{238}_{92}U$ may be written as

$$^{238}_{92}U \longrightarrow {}^{234}_{90}Th + {}^{4}_{2}He + Q$$

where Q (called Q-value) represents kinetic energy. Using *Einstein's mass-energy equivalence,* we get

$$Q = [m_N(^{238}_{92}U) - m_N(^{234}_{90}Th) - m_N(^{4}_{2}He)]\,c^2$$

By adding and subtracting 92 m_e in the bracket, we can write Q in terms of atomic masses as follows :

$$Q = [\{m_N(^{238}_{92}U) + 92\,m_e\} - \{m_N(^{234}_{90}Th) + 90 m_e\} - \{m_N(^{4}_{2}He) + 2\,m_e\}]\,c^2$$

$$= [238.05079 - (234.04363 + 4.00260)] \times 931.5 \text{ MeV} \quad [\because c^2 = 931.5 \text{ MeV / amu}]$$

$$= 0.00456 \times 931.5 \text{ MeV} = \mathbf{4.25\ MeV.}$$

(*b*) The kinetic energy of the α-particle,

$$E_\alpha \approx \left(\frac{A-4}{A}\right) Q = \frac{238-4}{238} \times 4.25 \text{ MeV}$$

$$= \mathbf{4.18\ MeV}$$

(*c*) If $^{238}_{92}U$ spontaneously emits a proton, the decay process would be

$$^{238}_{92}U \rightarrow {}^{237}_{91}Pa + {}^{1}_{1}H$$

For this process to happen, the Q-value will be

$$Q = [m(^{238}_{92}U) - m(^{238}_{92}Pa) - m(^{1}_{1}H)]\,c^2$$

$$= [238.05079 - 237.05121 - 1.00783] \text{ amu} \times c^2$$

$$= (-0.00825) \text{ amu} \times 931.5 \frac{\text{MeV}}{\text{amu}}$$

$$= \mathbf{-7.68\ MeV}$$

As the Q-value of the process is negative, it can not occur spontaneously.

Problems For Practice

1. Thorium $^{232}_{90}Th$ is converted into $^{208}_{89}Pb$ by radioactive transformations. How many α- and β-particles are emitted ? (Ans. 6 α and 4 β)
2. After a certain lapse of time, fraction of radioactive polonium undecayed is found to be 12.5% of the initial quantity. What is the duration of this time lapsed if the half life of polonium is 138 days ? (Ans. 414 days)

3. The half-life of ^{198}Au is 2.7 days. Calculate (*i*) the decay constant (*ii*) the average-life and (*iii*) the activity of 1.0 mg of ^{198}Au.
[Ans. (*i*) $2.9 \times 10^{-6} s^{-1}$, (*ii*) 3.9 days, (*iii*) 240 Ci]

4. Radon has 3.8 days as its half-life. How much radon will be left out of 15 mg mass after 38 days ?
(Ans. 0.015 mg)

5. Calculate the half life period of a radioactive substance if its activity drops to $\frac{1}{16}$ th of its initial value in 30 years. **[CBSE Sample Paper 11]**
(Ans. 7.5 years)

6. The half-life of polonium is 140 days. In what time will 15 g of polonium be disintegrated out of its initial mass of 16 ? (Ans. 560 days)

7. The half-life of radon is 3.8 days. After how many days will only one-twentieth of the sample be left over ? (Ans. 16.43 days)

8. 1 mg radium has 2.68×10^{18} atoms. Its half-life is 1620 years. How many radium atoms will disintegrate from 1 mg of pure radium in 3240 years ?
(Ans. 2.01×10^{18})

9. One gram of radium is reduced by 2 mg in 5 years by α-decay. Calculate the half-period of uranium **[Punjab 93]**
(Ans. 1671.7 years)

10. A nucleus of Ux_1 has a life of 24.1 days. How long a sample of Ux_1 will take to change to 90 % of it to Ux_2 ? (Ans. 80 days)

11. The atomic weight of radium is 226. It is observed that 3.67×10^{10} α-particles are emitted per second from one gram of radium. Calculate in years the half life period of radium. (Ans. 1595.7 years.)

12. Determine the amount of $^{210}_{54}$Po necessary to provide a source of α-particles of 5 mCi strength. The half-life of polonium is 138 days. (Ans. 1.11×10^{-6}g)

13. A radioactive element reduces to 25% of its mass in 1000 years. Find its half-life period ($\log_{10} 4 = 0.6021$). **[Punjab 2000, 03]**
(Ans. 499.86 years)

14. The half-life of a radioactive substance is 5×10^3 years. In many years its activity will decay to 0.2 times its initial value ? ($\log_{10} 5 = 0.6990$). **[Punjab 2000, 03]**
(Ans. 11615 years)

15. The half life of $^{238}_{92}$U undergoing α-decay is 4.5×10^9 years. Determine the activity of 10 g sample of $^{238}_{92}$U. Given that 1g of $^{238}_{92}$U contains 25.3×10^{20} atoms. **[CBSE OD 14C]**
(Ans. 1.23×10^5 disintegration/second)

16. Lanthanum has a stable isotope ^{139}La and radioactive isotope ^{138}La of half-life 1.1×10^{10} years whose atoms are 0.1% of those of the stable isotope. Estimate the rate of decay or activity of ^{138}La with 1 kg of ^{139}La. Take Avogadro's number, $N = 6 \times 10^{23}$ mol^{-1}. (Ans. 8600 s^{-1})

17. The count rate from a radioactive sample falls from $4.0 \times 10^6 s^{-1}$ to $1.0 \times 10^6 s^{-1}$ in 20 hours. What will be the count rate 100 hours after the beginning ?
(Ans. $3.91 \times 10^3 s^{-1}$)

18. The half life of a radioactive substance is 50 s. Calculate :
(*i*) The decay constant, and
(*ii*) Time taken for the sample to decay by 3/4th of the initial value. **[CBSE F 09]**
[Ans. (*i*) 0.01386 s, (*ii*) 100 s]

HINTS

1. Proceed as in Example 12 on page 13.22.

2. $N = 12.5\%$ of $N_0 = \frac{12.5}{100} N_0 = \frac{1}{8} N_0$

As $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n \quad \therefore \quad \frac{1}{8} = \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n$ or $n = 3$

Time of disintegration $= 3 \times 138 =$ **414 days.**

3. (*i*) Decay constant, $\lambda = \frac{0.693}{T_{1/2}}$

$$= \frac{0.693}{2.7 \text{ days}} = \frac{0.693}{2.7 \times 24 \times 3600 \text{ s}}$$

$$= \mathbf{2.9 \times 10^{-6} s^{-1}}.$$

(*ii*) Average-life,

$$\tau = \frac{1}{\lambda} = \frac{2.7}{0.693} = \textbf{3.9 days.}$$

(*iii*) Number of atoms in 1.0 mg (or 1.0×10^{-3} g) of Au-198 is

$$N = \frac{6.023 \times 10^{23}}{198} \times 1.0 \times 10^{-3} = 3.04 \times 10^{18}$$

Activity,

$$R = \lambda N = 2.9 \times 10^{-6} \times 3.04 \times 10^{18}$$

$$= 8.8 \times 10^{12} \text{ disintegrations / s}$$

$$= \frac{8.8 \times 10^{12}}{3.7 \times 10^{10}} \text{ Ci} = \textbf{240 Ci.}$$

4. Here $n = \frac{t}{T_{1/2}} = \frac{38}{3.8} = 10$

$$N = N_0 \left(\frac{1}{2}\right)^n = 15 \text{ mg} \times \left(\frac{1}{2}\right)^{10}$$

$$= \frac{15}{1024} = \textbf{0.015 mg.}$$

5. As activity ∝ number of atoms present, so

$$N = \frac{N_0}{16}$$

But $N = N_0\left(\frac{1}{2}\right)^n$

$\therefore \quad \frac{N_0}{16} = N_0\left(\frac{1}{2}\right)^n$ or $\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n$ or $n = 4$

$$\text{Half-life period} = \frac{\text{Time of disintegration}}{\text{Number of half lives}}$$

$$= \frac{30}{4} = \mathbf{7.5\ years}$$

6. $N_0 = 16\,g$, $N = 16 - 15 = 1\,g$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

$\therefore \quad \frac{1}{16} = \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n$ or $n = 4$

Time of disintegration

$$= T_{1/2} \times n = 140 \times 4 = \mathbf{560\ days.}$$

7. Here $T_{1/2} = 3.8$ days, $N = \frac{N_0}{20}$

$\therefore$ Disintegration constant,

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.8}\ \text{day}^{-1}$$

As $N = N_0 e^{-\lambda t}$

$\therefore \quad \frac{N_0}{20} = N_0\, e^{-\lambda t}$ or $e^{\lambda t} = 20$

Taking natural logarithm,

$\lambda t \log_e e = \log_e 20$ or $\lambda t = 2.303 \times \log_{10} 20$

or $\quad t = \frac{2.303 \times 1.3010}{\lambda}$

$$= \frac{2.303 \times 1.3010 \times 3.8}{0.693}\ \text{days} = \mathbf{16.43\ days.}$$

8. $n = \frac{t}{T_{1/2}} = \frac{3240}{1620} = 2$

Mass of radium left after 2 half-lives is

$$N = N_0\left(\frac{1}{2}\right)^n = 1 \times \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25\ \text{mg}$$

Mass of radium disintegrated = 1 − 0.25 = 0.75 mg

Number of radium atoms disintegrated

$$= 0.75 \times 2.68 \times 10^{18} = \mathbf{2.01 \times 10^{18}}.$$

9. Here $m = 1\,g$, $m_0 - m = 2\ \text{mg} = 0.002\ g$

$\therefore \quad m = m_0 - 0.002 = 1 - 0.002 = 0.998\ g$

As $\quad m = m_0\, e^{-\lambda t}$

$\therefore \quad e^{\lambda t} = \frac{m_0}{m}$ or $\lambda t = \log_e \frac{m_0}{m}$

or $\quad \lambda = \frac{2.303}{t}\log_{10}\frac{m_0}{m} = \frac{2.303}{5}\log_{10}\frac{1}{0.998}$

$$= \frac{2.303}{5}\log_{10} 1.002 = \frac{2.303 \times 0.0009}{5}$$

$$= 4.1454 \times 10^{-4}\ \text{year}^{-1}$$

$\therefore \quad T = \frac{0.693}{\lambda} = \frac{0.693}{4.1454 \times 10^4}$

$$= \mathbf{1671.7\ years}$$

10. Here $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{24.1} = 0.02876\ \text{day}^{-1}$;

$$N = N_0 - 90\%\ \text{of}\ N_0 = \frac{N_0}{10}$$

As $\quad N = N_0 e^{-\lambda t} \quad \therefore \quad \frac{N_0}{10} = N_0 e^{-\lambda t}$

or $\quad \frac{1}{10} = e^{-\lambda t}$ or $10 = e^{\lambda t}$

or $\log_e 10 = \lambda t$

$\therefore \quad t = \frac{1}{\lambda}\log_e 10 = \frac{2.303 \log 10}{0.02870}$

$$= \frac{2.303 \times 1}{0.02876} \simeq \mathbf{80\ days.}$$

11. Here $\frac{dN}{dt} = 3.67 \times 10^{10}$ disintegrations/s

$$N = \frac{\text{Avogadro number}}{\text{Atomic mass}} = \frac{6.023 \times 10^{23}}{226}$$

As $\quad \frac{dN}{dt} = \lambda N$

$$3.67 \times 10^{10} = \lambda \times \frac{6.023 \times 10^{23}}{226}$$

or $\quad \lambda = \frac{3.67 \times 10^{10} \times 226}{6.023 \times 10^{23}}\ \text{s}^{-1}$

Hence $\quad T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693 \times 6.023 \times 10^{23}}{3.67 \times 10^{10} \times 226}\ \text{s}$

$$= \frac{0.693 \times 6.023 \times 10^{23}}{3.67 \times 10^{10} \times 226 \times 60 \times 60 \times 24 \times 365}$$

$$= \mathbf{1595.7\ years.}$$

12. The decay rate is given by

$$R = \left|\frac{dN}{dt}\right| = \lambda N = \frac{0.693}{T_{1/2}} \times N$$

Given $R = 5\ \text{mCi} = 5 \times 3.7 \times 10^7\ \text{dis s}^{-1}$

$T_{1/2} = 138\ \text{days} = 138 \times 8.64 \times 10^4\ \text{s}$

$$= 1.192 \times 10^7\ \text{s}$$

$$N = \frac{RT_{1/2}}{0.693} = \frac{5 \times 3.7 \times 10^7 \times 1.192 \times 10^7}{0.693}\ \text{atoms}$$

$$= 3.18 \times 10^{15}\ \text{atoms}$$

As 210 g of polonium contains 6.023×10^{23} atoms, therefore, the amount necessary to obtain a source of the required strength

$$= \frac{210 \times 3.18 \times 10^{15}}{6.023 \times 10^{23}}\ g = \mathbf{1.11 \times 10^{-6}\ g.}$$

13. Here $m = 25\%$ of $m_0 = m_0/4$, $t = 1000$ years

From Problem 9,

$$\lambda = \frac{2.303}{t}\log_{10}\frac{m_0}{m} = \frac{2.303}{1000}\log_{10}\frac{m_0}{m_0/4}$$

$$= \frac{2.303}{1000}\log_{10}4 = \frac{2.303 \times 0.6021}{1000}$$

$$= 1.3866 \times 10^{-3}\ \text{year}^{-1}.$$

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{1.3866 \times 10^{-3}} = \mathbf{499.86\ years}$$

14. Here $R = 0.2\ R_0$ or $R_0/R = 5$

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5 \times 10^3}$$

As $\quad R = R_0\, e^{-\lambda t}$

$$\therefore \quad t = \frac{2.303}{\lambda}\log_{10}\frac{R_0}{R}$$

$$= \frac{2.303 \times 5 \times 10^3}{0.693}\log_{10}5$$

$$= \frac{2.303 \times 5 \times 10^3 \times 0.6990}{0.693} = \mathbf{11615\ years}$$

15. $T_{1/2} = 4.5 \times 10^9$ years $= 4.5 \times 10^9 \times 3.156 \times 10^7$ s

$N = 10 \times 25.3 \times 10^{20}$ atoms

$$R = \lambda N = \frac{0.693}{T_{1/2}}.N = \frac{0.693 \times 253 \times 10^{20}}{4.5 \times 3.156 \times 10^{16}}$$

$= \mathbf{1.23 \times 10^5}$ **disintegrations/second.**

16. Number of atoms in 1 kg (or 1000 g) of ^{139}La

$$= \frac{m}{M} \times \text{Avogadro's number} = \frac{1000 \times 6 \times 10^{23}}{139}$$

As the given sample contains 0.1% (or 10^{-3} times) ^{138}La atoms, therefore,

Number of atoms of ^{138}La,

$$N = \frac{10^{-3} \times 10 \times 6 \times 10^{23}}{139} = \frac{6 \times 10^{23}}{139}$$

Disintegration constant,

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.1 \times 10^{10}}\ \text{year}^{-1}$$

$$= \frac{0.693}{1.1 \times 10^{10} \times 365 \times 24 \times 3600}\ \text{s}^{-1}$$

$\therefore$ Rate of decay,

$$R = \left|\frac{dN}{dt}\right| = \lambda N$$

$$= \frac{0.693 \times 6 \times 10^{23}}{1.1 \times 10^{10} \times 365 \times 24 \times 3600 \times 139}$$

$$= \mathbf{8600\ s^{-1}}.$$

17. Here $R_0 = 4.0 \times 10^6\,\text{s}^{-1}$, $R = 1.0 \times 10^6\,\text{s}^{-1}$, $t = 20$ hours

Now $\quad \dfrac{R}{R_0} = \left(\dfrac{1}{2}\right)^n \quad$ or $\quad \dfrac{1.0 \times 10^6}{4.0 \times 10^6} = \left(\dfrac{1}{2}\right)^n$

or $\quad \left(\dfrac{1}{2}\right)^2 = \left(\dfrac{1}{2}\right)^n \quad \therefore \quad n = 2$

$$T_{1/2} = \frac{t}{n} = \frac{20}{2} = 10\ \text{hours}$$

Now $T' = 100$ hours, $\quad n' = \dfrac{t}{T_{1/2}} = \dfrac{100}{10} = 10$

$$\therefore \quad R' = R_0\left(\frac{1}{2}\right)^{n'} = 4.0 \times 10^6\left(\frac{1}{2}\right)^{10} = \mathbf{3.91 \times 10^3\ s^{-1}}.$$

18. (*i*) $\lambda = \dfrac{0.693}{50} = \mathbf{0.01386\ s}.$

(*ii*) In two half lives, the number reduces by $\left(\dfrac{1}{2}\right)^2$ or $\dfrac{3}{4}$-th of the initial value.

13.26 NATURAL AND ARTIFICIAL RADIOACTIVITY

30. *Distinguish between natural and artificial radioactivity.*

Natural radioactivity. *The phenomenon of the spontaneous emission of* α-, β- or γ-radiations from the nuclei of naturally occurring isotopes is called natural radioactivity.

Artificial or induced radioactivity. *The phenomenon of inducing radioactivity in certain stable nuclei by bombarding them by suitable high energy particles is called artificial or induced radioactivity.*

Examples of induced radioactivity :

1. When boron is bombarded by α-particles, it forms a radioactive isotope of nitrogen which decays into carbon with a half-life of 10.1 minutes.

$$^{10}_{5}B + ^{4}_{2}He \longrightarrow ^{13}_{7}N + ^{1}_{0}n$$

$$^{13}_{7}N \longrightarrow ^{13}_{6}C + ^{0}_{1}e + \nu$$

2. When aluminium is bombarded by α-particles, it produces a radioactive phosphorus which decays into silicon with a half life of 2.55 minutes.

$$^{27}_{13}Al + ^{4}_{2}He \longrightarrow ^{30}_{15}P + ^{1}_{0}n$$

$$^{30}_{15}P \longrightarrow ^{30}_{14}Si + ^{0}_{1}e + \nu$$

13.27 NUCLEAR REACTION

31. *What is a nuclear reaction ? Give an example.*

Nuclear reaction. *A reaction which involves the change of stable nucleus of one element into the nucleus of another* element is called nuclear reaction. It is usually caused by bombarding the reacting species with suitable high energy particles.

Getting a clue from the spontaneous disintegration of radioactive nuclei, *Rutherford* thought that it might be possible to penetrate heavy nucleus with a high-speed particle such as α-particle and thereby either produce a nucleus with a greater mass number or induce an artificial disintegration of the nucleus. In 1919, *Rutherford* succeeded in bringing out the first artificial transmutation by bombarding nitrogen nuclei with α-particles which produced an isotope of oxygen and a proton according to the reaction :

$$^{14}_{7}N + ^{4}_{2}He \longrightarrow ^{17}_{8}O + ^{1}_{1}H \qquad ...(1)$$

Such an artificial nuclear transformation is termed as a nuclear reaction. Nuclear reactions can be used for the production of newer nuclei or for energy generation.

32. *How is a nuclear reaction represented symbolically ? What is Q-value of nuclear reaction ? State the conservation laws which are obeyed in nuclear reactions.*

Symbolic representation of a nuclear reaction. In general, in a nuclear reaction there is a collision between a ***target nucleus** A* and ***bombarding particle*** or ***projectile a*** which produces another nucleus *B* and an elementary particle *b* or gamma ray. This reaction may be represented as

$$A + a \longrightarrow B + b + Q \qquad ...(2)$$

The factor *Q* represents the energy absorbed or released in the nuclear reaction. This is called ***Q-value*** or ***nuclear reaction energy***. *It is equal to the difference between the sum of the rest masses of reacting particles and the sum of the rest masses of the product particles.* In fact, it is equal to the total change in kinetic energy of the system.

The nuclear reaction represented by equation (2) can be expressed in a compact notation devised by *Bethe* :

$$A(a, b)\, B$$

So we can represent the reaction of equation (1) as follows :

$$N(\alpha, p)\, O$$

If the initial rest mass is greater than the final rest mass, then the *Q*-value of the reaction is ***positive***. Such a nuclear reaction is called exothermic or ***exoergic reaction***. The energy released in the reaction can be harnessed as a source of energy, called ***nuclear energy***. This energy appears in the form of kinetic energy of the product particles.

If the initial rest mass is less than the final rest mass, the *Q*-value of the reaction is ***negative***. Such a reaction is called ***endothermic*** or ***endoergic reaction***. This much energy is absorbed in the reaction. For such a reaction to take place, the required energy has to be supplied in the form of kinetic energy of the bombarding particle.

The energy released or absorbed in a nuclear reaction can be calculated from the Einstein's mass-energy equivalence relation :

$$E = \Delta m \,.\, c^2$$

In any nuclear reaction, the following quantities are *conserved* :

1. **Momentum**. The total momentum of the particles entering into the reaction is equal to the total momentum of the products after the reaction.
2. **Nucleons**. The total number of nucleons before and after the reaction remains the same.
3. **Charge.** The total charge of the product particles is equal to that of the reactant particles.
4. **Energy**. The total energy (kinetic energy + rest mass energy) of the product particles is equal to the total energy of the reactant particles.

13.28 A NUCLEAR REACTION VS. A CHEMICAL REACTION

33. *How does a nuclear reaction differ from a chemical reaction ?*

A nuclear reaction *vs.* a chemical reaction. A nuclear reaction differs markedly from a chemical reaction. In a chemical reaction, only the electrons revolving around the nucleus take part in the reaction and no change occurs inside the nucleus whereas in a nuclear reaction, the nucleus itself undergoes a transformation. The energy changes involved in chemical reactions are much smaller than the energy changes involved in nuclear reactions.

13.29 TYPES OF NUCLEAR REACTIONS

34. *Name the different projectiles commonly used in nuclear reactions. How do we classify nuclear reactions on the basis of projectile particle and outgoing particle ? Give examples of each type.*

Types of nuclear reactions. In nuclear reactions, the bombarding particles or projectiles generally used are α-particle ($^{4}_{2}He$), proton ($^{1}_{1}H$), deuteron ($^{2}_{1}H$), neutron ($^{1}_{0}n$) and gamma ray photon (γ). Depending on the nature of the projectile particle and the outgoing particle, the various nuclear reactions are classified as follows :

1. Alpha particle-induced reactions :

(i) ${}^{10}_{5}B + {}^{4}_{2}He \rightarrow {}^{13}_{6}C + {}^{1}_{1}H$ — **(α, p) reaction**

(ii) ${}^{9}_{4}Be + {}^{4}_{2}He \rightarrow {}^{12}_{6}C + {}^{1}_{0}n$ — **(α, n) reaction**

2. deuteron-induced reactions :

(i) ${}^{12}_{6}C + {}^{2}_{1}H \rightarrow {}^{13}_{7}N + {}^{1}_{0}n$ — **(d, n) reaction**

(ii) ${}^{16}_{8}O + {}^{2}_{1}H \rightarrow {}^{14}_{7}N + {}^{4}_{2}He$ — **(d, α) reaction**

(iii) ${}^{7}_{3}Li + {}^{2}_{1}H \rightarrow {}^{8}_{3}Li + {}^{1}_{1}H$ — **(d, p) reaction**

3. Proton-induced reactions :

(i) ${}^{14}_{7}N + {}^{1}_{1}H \rightarrow {}^{15}_{8}O + \gamma$ — **(p, γ) reaction**

(ii) ${}^{7}_{3}Li + {}^{1}_{1}H \rightarrow {}^{4}_{2}He + {}^{4}_{2}He$ — **(p, α) reaction**

(iii) ${}^{18}_{8}O + {}^{1}_{1}H \rightarrow {}^{18}_{9}F + {}^{1}_{0}n$ — **(p, n) reaction**

(iv) ${}^{9}_{4}Be + {}^{1}_{1}H \rightarrow {}^{8}_{4}Be + {}^{2}_{1}H$ — **(p, d) reaction**

4. Neutron-induced reactions :

(i) ${}^{23}_{11}Na + {}^{1}_{0}n \rightarrow {}^{20}_{9}F + {}^{4}_{2}He$ — **(n, α) reaction**

(ii) ${}^{14}_{7}N + {}^{1}_{0}n \rightarrow {}^{14}_{6}C + {}^{1}_{1}H$ — **(n, p) reaction**

(iii) ${}^{238}_{92}U + {}^{1}_{0}n \rightarrow {}^{239}_{92}U + \gamma$ — **(n, γ) reaction**

(iv) ${}^{131}_{52}Te + {}^{1}_{0}n \rightarrow {}^{132}_{53}I + {}^{0}_{-1}e$ — **(n, β) reaction**

5. Gamma ray photon-induced reactions :

Such reactions are also called ***photo-nuclear reactions*** or ***photo disintegrations***.

${}^{31}_{15}P + \gamma \rightarrow {}^{30}_{15}P + {}^{1}_{0}n$ — **(γ, n) reaction**

${}^{25}_{12}Mg + \gamma \rightarrow {}^{24}_{11}Na + {}^{1}_{1}H$ — **(γ, p) reaction**

13.30 ENERGY FROM THE NUCLEUS

35. *What is nuclear energy ? With the help of the binding energy curve, explain how nuclear energy can be released.*

Nuclear energy. *The energy released during a nuclear reaction is called nuclear energy.* As shown in Fig. 13.5, the curve of binding energy per nucleon has long flat region in the mass number range from 30 to 170. Here the binding energy per nucleon is almost constant, around 8.5 MeV. However, for $A < 30$ and $A > 170$, B.E./nucleon is less than this plateau-value. This indicates that these nuclei are less tightly bound or less stable than the nuclei with mass numbers between 30 and 170. Hence whenever an element with a smaller binding energy is transmuted into an element with a larger binding energy, a tremendous amount of energy is released. This is due to the conversion of some mass into energy in accordance with Einstein's mass-energy relation. The energy released in a nuclear reaction due to the decrease in mass Δm is given by

$$Q = -\Delta m \cdot c^2$$

This equation indicates that even a very small quantity of matter is capable of releasing very large amount of energy. The nuclear reactions which can be exploited to produce energy are of *two* broad types :

1. **Nuclear fission** in which a heavy nucleus splits up into two smaller nuclei, liberating a large amount of energy as in an atom bomb.

2. **Nuclear fusion** in which two smaller nuclei fuse together to form a larger nucleus, releasing a large amount of energy as in a hydrogen bomb.

13.31 NUCLEAR FISSION

36. *What is nuclear fission ? Mention the different ways in which a uranium nucleus can undergo fission. How is the probability of occurrence of these fissions related to the mass number of the nuclides formed ? Explain it with the help of a suitable graph.*

Nuclear fission. *The phenomenon in which a heavy nucleus ($A > 230$) when excited splits into two smaller nuclei of nearly comparable masses is called nuclear fission.*

In 1938, German scientists *Otto Hahn* and *Fritz Strassmann* found that when uranium is bombarded by slow moving neutrons, a ${}^{235}_{92}U$ nucleus gets excited by capturing a slow moving neutron and splits into two nearly equal fragments like ${}^{141}_{56}Ba$ and ${}^{92}_{36}Kr$ alongwith the emission of 3 neutrons. The nuclear reaction involved can be written as

$${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n + Q$$

The Q-value of this reaction is about 200 MeV.

Fission products of uranium. The detailed analysis of the products of fission of ${}^{235}_{92}U$ has revealed that this fission, in general, produces nuclei with mass numbers in the range 72 (${}_{30}Zn$) to 158 (${}_{63}Eu$). This means that nuclei of different mass numbers can be produced by fission of uranium. For example,

$${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2\,{}^{1}_{0}n$$

$${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{95}_{42}Mo + {}^{139}_{57}La + 2\,{}^{1}_{0}n$$

$${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{137}_{53}I + {}^{97}_{39}Y + 2\,{}^{1}_{0}n$$

$${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{144}_{55}Cs + {}^{90}_{37}Pb + 2\,{}^{1}_{0}n$$

A number of other combinations are formed. The two nuclides have generally unequal mass numbers. If the relative yield of different nuclides are plotted against their mass number, we get a plot of the type shown in Fig. 13.14.

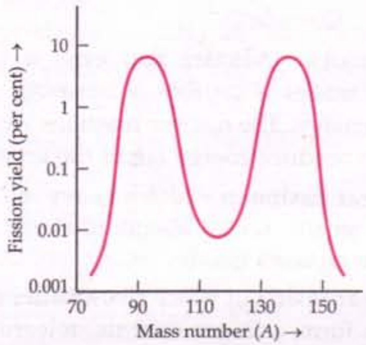

Fig. 13.14 Distribution by mass number of fragments formed due to fission of ^{235}U nuclei.

The fission of ^{235}U yields broadly two groups of nuclei. One of the groups is a '*light group*' with mass number range from 85 to 104. The other group is a '*heavy group*' with mass number range from 130 to 149. The most probable fissions, having about 7% chance are centred around $A = 95$ and $A = 140$. The chances of fission with two fragments of equal mass number ($A = 117$) are extremely small, only about 0.01%.

Generally, the fission nuclides contain an excess of neutrons and are, therefore, highly unstable. They undergo beta decay until they form stable end products.

The decay chain for Xe is

$$^{140}_{54}Xe \xrightarrow{\beta^-} {}^{140}_{55}Cs \xrightarrow{\beta^-} {}^{140}_{56}Ba \xrightarrow{\beta^-} {}^{140}_{57}La \xrightarrow{\beta^-} {}^{140}_{58}Ce$$

For strontium, the decay chain is

$$^{94}_{38}Sr \xrightarrow{\beta^-} {}^{94}_{39}Y \xrightarrow{\beta^-} {}^{94}_{40}Zr$$

13.32 THEORY OF NUCLEAR FISSION

37. *Explain nuclear fission on the basis of liquid drop model of the nucleus.*

Explanation of nuclear fission on the basis of liquid drop model. In 1939, *Yakov Frankel, Niels Bohr* and *John A. Wheeler* proposed the liquid drop model to explain fission of nuclei. In this model, the nucleus is assumed as a liquid drop of spherical shape, which is incompressible and has a uniform positive charge. The repulsion between the positively charged protons tends to split the nucleus. But the forces of surface tension and short range nuclear forces hold the nucleons together. When a nucleus captures a neutron as shown in Fig. 13.15, its equilibrium is disturbed and it begins to oscillate about its spherical shape due to the energy of the absorbed neutron. When the excitation energy is sufficiently large, the compound nucleus deforms into a dumb-bell shape structure. The oscillations eventually lead to the fission of the nucleus into two fragments, accompanied by emission of neutrons. The nuclei of the fission products are also unstable, they attain stability by emitting β-rays, γ-rays and more neutrons.

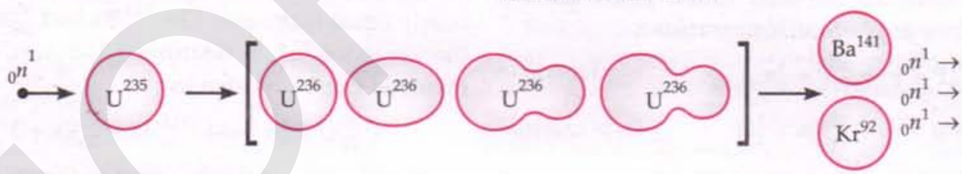

Fig. 13.15 Nuclear fission on the basis of liquid drop model.

13.33 NUCLEAR FISSION AS A SOURCE OF ENERGY

38. *Explain how is nuclear fission an enormous source of energy.*

Nuclear fission as a source of energy. In a nuclear fission, the sum of the masses of the final products is less than the sum of the masses of the reactant components. The difference in masses, called *mass defect*, is converted into energy according to Einstein's mass-energy relationship ($E = mc^2$). Thus an enormous amount of energy is released in a nuclear fission, as can be seen from the following example :

$$^{235}_{92}U + {}^1_0n \longrightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^1_0n + Q$$

Initial masses		Final masses	
$^{235}_{92}U$	235.0439 amu	$^{141}_{56}Ba$	140.9139 amu
^1_0n	1.0087 amu	$^{92}_{36}Kr$	91.8973 amu
		$3\,^1_0n$	3.0261 amu
	236.0526 amu		235.8373 amu

Mass defect $= 236.0526 - 235.8373 = 0.2153$ amu

As 1 amu $= 931$ MeV

$\therefore$ Energy released, $Q = 0.2153 \times 931$ MeV

$\simeq$ **200 MeV**

This energy appears in the form of kinetic energy of the fission products and as γ-rays.

Thus in the fission of a single nucleus of $^{235}_{92}U$, about 200 MeV energy is released which is equivalent to 0.9 MeV/nucleon. The total energy released in the fission of 1 kg of naturally occurring uranium, which contains about 2.56×10^{24} atoms of $^{235}_{92}U$ isotope, will be $200 \times 2.56 \times 10^{24}$ MeV $= 10^{14}$ J. This is a very huge amount of energy which is equal to the energy obtained by burning of 3 tons of coal.

13.34 NUCLEAR CHAIN REACTION

39. *Explain how a chain reaction can occur in a fissionable material. What are uncontrolled and controlled chain reactions ?*

Nuclear chain reaction. Nuclear fission is a peculiar type of reaction which, besides the other fission products, produces the same kind of particles that initiate it, *viz.*, neutrons.

When a single $^{235}_{92}U$ nucleus captures a neutron, its fission produces 2.5 neutrons. These freshly produced electrons can further cause the fission of more uranium nuclei, producing still more neutrons, which can further cause the fission of a larger number of nuclei, and so on. *The number of fissions taking place at each successive stage goes on increasing at a rapid rate* (*rather in a geometric progression*). *Thus a chain reaction is set up*, as illustrated in Fig. 13.16. *Enrico Fermi* first suggested the possibility of such a reaction in 1939.

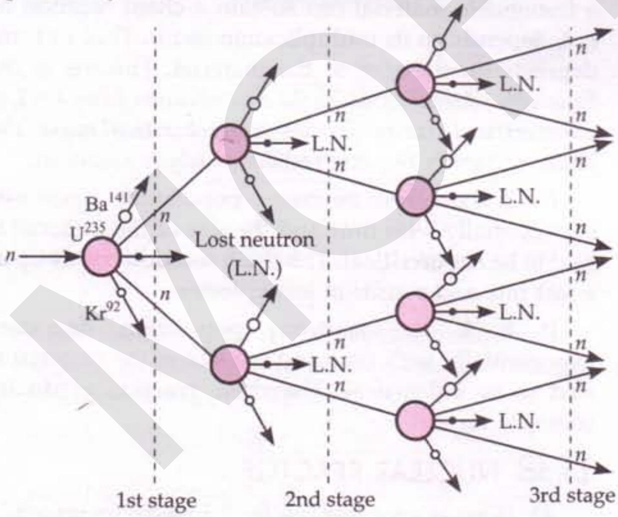

Fig. 13.16 A chain reaction in $^{235}_{92}U$.

Uncontrolled chain reaction. If a chain reaction is started in a fissionable material having mass greater than certain critical mass, then the reaction will accelerate at such a rapid rate that the whole material will explode within a microsecond, liberating a huge amount of energy. Such a chain reaction is called *uncontrolled chain reaction*. It forms the underlying principle of the *atomic bombs*.

Controlled chain reaction. The chain reaction can be controlled and maintained steady by absorbing a suitable number of neutrons at each stage of the reaction, so that on an average one neutron remains available for exciting further fission. Such a reaction is called controlled chain reaction. Here the energy released does not get out of control. A *nuclear reactor* works on the principle of a controlled chain reaction.

The first successful experiments to control fission were made by *Enrico Fermi* and his co-workers in University of Chicago, USA, in 1942.

13.35 THERMAL NEUTRONS AND MODERATOR

40. *What are thermal neutrons ? Explain the action of a moderator in slowing down the fast moving neutrons.*

Thermal neutrons. The neutrons produced in fission of ^{235}U nuclei have average kinetic energy ≈ 2 MeV. Such neutrons are called ***fast neutrons***. The ^{235}U nuclei have good probability of absorbing slow neutrons of thermal energy ≈ 0.0235 eV, but have poor chance of absorbing fast neutrons. Unless slowed down to thermal energies, the fast neutrons will escape from the fissionable material without causing any fission. *The slow moving neutrons of energies* 0.0235 *eV are called* ***thermal neutrons***. These neutrons have velocities $\approx 2200 \text{ ms}^{-1}$, which are the random velocities of atoms and molecules at room temperature.

Moderator. *Any substance which is used to slow down fast moving neutrons to thermal energies* (≈ 0.0235 *eV*) *is called a moderator*. The commonly used moderators are water, heavy water (D_2O), graphite and beryllium.

Action of moderator. Fast neutrons are passed through substances like paraffin, deuterium or water, which contain large number of hydrogen nuclei or protons. Neutrons and protons have nearly the same mass. When fast moving neutrons are passed through paraffin, they make elastic collisions with protons, which have comparatively much smaller velocities. In few interactions, the velocities of the neutrons get interchanged with those of protons. The final velocities of the neutrons correspond to the random velocities of the atoms or molecules of the moderator at the room temperature. Such neutrons are called *thermal neutrons*.

About 25 collisions with deuterons (present in heavy water) or 100 collisions with carbon or beryllium are sufficient to slow down a neutron from 2 MeV to thermal energies.

A good moderator has *two* properties :

1. It slows down neutrons by elastic collisions.
2. It does remove neutrons from the system by absorbing them.

13.36 DIFFICULTIES IN SUSTAINING A CHAIN REACTION

41. *Mention the difficulties encountered in sustaining a chain reaction in a nuclear reactor. Or, explain why does a chain reaction die out.*

Difficulties in sustaining a chain reaction. To ensure high probability of chain reaction, it is desirable to have sufficient concentration of ^{235}U in the uranium being used in fission reactions. Natural uranium consists of three isotopes : ^{234}U, ^{235}U and ^{238}U with the following compositions :

$$^{234}U = 0.0058\%$$
$$^{235}U = 0.715\%,$$
and $$^{238}U = 99.28\%$$

The concentration of ^{235}U can be increased by special techniques. This process is called ***enrichment*** and the processed uranium is called ***enriched uranium,*** which contains about 3% ^{235}U.

The probability of fission in the enriched uranium is very high. Still there are a number of difficulties due to which a chain reaction dies out.

1. **Neutron leakage.** Some of the neutrons produced by fission may not interact with other nuclei and escape from the system. But leakage is a surface effect. The fraction of neutrons lost by leakage can be made sufficiently small by making the fissionable system large enough, thereby reducing the surface-to-volume ratio.

2. **Neutron energy.** Neutrons produced in fission are fast neutrons, having kinetic energies of about 2 MeV. Such neutrons have more chances of escaping from the fissionable material, without causing further fission. So these neutrons are slowed down to thermal energies by mixing suitable moderator with the uranium fuel.

3. **Neutron capture.** When fast neutrons (2 MeV) produced by fission are slowed down in a moderator to thermal energies (≈ 0.04 eV), they pass through a critical energy range 1 – 100 eV. The neutrons of this energy range have good chances of being absorbed by ^{238}U nuclei which are present in large numbers ($\approx 99.3\%$). This *resonance capture* is non-fissionable and results in the emission of a γ-ray. In this way, many neutrons get removed from the fission chain. To minimise this non-fissionable capturing, uranium fuel and moderator are not intimately mixed, instead they are placed in alternate columns. The distances between the fuel rods are so adjusted that a neutron coming from one rod is slowed down to thermal energies before it enters the neighbouring rod. This eliminates the possibility of non-fission capture of the neutrons.

13.37 MULTIPLICATION FACTOR AND CRITICAL SIZE

42. *Define multiplication factor and critical size for a fissionable mass. Give their physical significance.*

Multiplication factor. Whether a chain reaction, once started in a fissionable mass, will remain steady, increase or decrease, depends on a parameter called multiplication factor. *The multiplication factor of a fissionable mass is defined as the ratio of the number of neutrons present at the beginning of a particular generation to the number of neutrons present at the beginning of the previous generation.* Thus

$$k = \frac{\text{Number of neutrons present at the beginning of one generation}}{\text{Number of neutrons present at the beginning of previous generation}}$$

The multiplication factor k gives a measure of the growth rate of the neutrons in a fissionable mass.

If $k > 1$, the chain reaction *grows.*

If $k = 1$, the chain reaction remains *steady.*

If $k < 1$, the chain reaction gradually *dies* out.

Critical size and critical mass. Whether the mass of a fissionable material can sustain a chain reaction or not, depends on its multiplication factor. This, in turn, depends on the size of the material. The *size of the fissionable material for which the multiplication factor k = 1, is called* ***critical size*** *and its mass is called* ***critical mass****. The chain reaction in this case remains steady or sustained.*

If $k > 1$, the neutron population increases exponentially with time and the size of the material is said to be **supercritical.** The chain reaction builds up at a fast rate and results in an explosion.

If $k < 1$, the neutrons population decreases exponentially with time and the size of the material is said to be **subcritical**. The chain reaction gradually comes to an end.

13.38 NUCLEAR REACTOR

43. *What is a nuclear reactor ? Explain its principle, construction and working. State some of the important uses of nuclear reactors.*

Nuclear reactor. *It is a device in which a nuclear chain reaction is initiated, maintained and controlled.* It works on the principle of controlled chain reaction and provides energy at a constant rate.

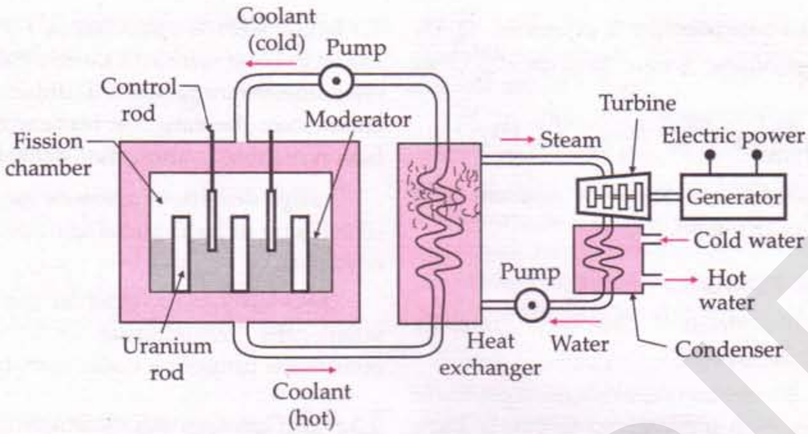

Fig. 13.17 Nuclear reactor.

Main parts of a nuclear reactor :

1. **Nuclear fuel.** It is the material that can be fissioned by neutrons. The isotopes like U-235, Th-232 and Pu-239 can be used as the reactor fuel. A certain mass of the fuel is taken in the form of rods, tightly sealed in aluminium containers. The rods, separated by moderator, are placed in the *core* of the reactor.

2. **Moderator.** In the fission of uranium, fast neutrons of energy 2 MeV are released. These fast neutrons have more tendency to escape instead of triggering another fission reaction. Also, slow neutrons are more efficient in inducing fission in $^{235}_{92}U$ nuclei than fast neutrons. By the use of a moderator, the fast neutrons are slowed to thermal velocities. Usually, heavy water, graphite and beryllium oxide are used as moderators.

3. **Control rods.** To start, stop or control the chain reaction, rods of neutron absorbing material like cadmium or boron are inserted into the reactor core. The rate of neutron production is controlled by adjusting the depth of control rods.

4. **Coolant.** It is the material used to cool the fuel rods and the moderator and is capable of carrying away large amount of heat produced in the fission process. The coolant transfers heat to the working liquid like water and produces steam. The steam drives a turbine which, in turn, runs a generator to generate electric power. *The coolant must have high boiling point and high specific heat.* Heavy water and liquid sodium are good coolants.

5. **Shielding.** The intense neutrons and gamma radiations produced in nuclear reactor are harmful for human body. To protect the workers from these radiations, the reactor core is surrounded by a thick concrete wall, called the *reactor shield.*

Working. Initially, some neutrons are produced by the action of α-particles on polonium or beryllium. They are slowed down and are used to start fission of $^{235}_{92}U$ nuclei. Fast neutrons are released in these fissions which are slowed down to thermal velocities by passing them through the moderator. These slow neutrons cause fission of more $^{235}_{92}U$ nuclei and thus the chain reaction builds up. By raising or lowering the control rods, the chain reaction is suitably controlled.

Uses of nuclear reactor :

1. In the preparation of radio-isotopes, which find extensive use in scientific research, medicine, agriculture and in industry.
2. In the generation of electric power.
3. In the production of fast neutrons which are needed in nuclear bombardment.
4. In producing fissile material like plutonium which is used in atomic bombs.

13.39 BREEDER REACTORS

44. *What are breeder reactors ?*

Breeder reactors. *A breeder reactor is one that produces more fissionable nuclei than it consumes.*

Natural uranium contains very little (0.7%) of the fissile ^{235}U isotope. Its contains mostly (99.3%) of the non-fissile ^{238}U isotope. When this isotope is bombarded with fast neutrons, the following nuclear transmutation occurs :

$$^{238}_{92}U + ^{1}_{0}n \longrightarrow ^{239}_{92}U \xrightarrow{-\beta^-} ^{239}_{93}Np \xrightarrow{-\beta^-} ^{239}_{94}Pu$$

Thus the breeder reactor produces fissile plutonium $^{239}_{94}Pu$ from non-fissile uranium. Similarly, naturally more abundant isotope of thorium, ${}^{232}_{90}\text{Th}$, can be used to produce fissile isotope ${}^{233}_{92}\text{U}$ as follows :

$$ {}^{232}_{90}\text{Th} + {}^{1}_{0}n \longrightarrow {}^{233}_{90}\text{Th} \xrightarrow{-\beta^-} {}^{233}_{91}\text{Pa} \xrightarrow{-\beta^-} {}^{233}_{92}\text{U} $$

In breeder reactors, an alloy of sodium and potassium is used as a coolant.

13.40 NUCLEAR FUSION

45. *What is nuclear fusion ? Give some examples. What is thermonuclear fusion ?*

Nuclear fusion. *The process in which two light nuclei combine (at extremely high temperature) to form a single heavier nucleus is called nuclear fusion.*

The mass of the heavier nucleus formed is less than the sum of the masses of the combining nuclei. The *mass defect* is released as energy in accordance with Einstein's mass-energy relation $E = \Delta m \,.\, c^2$. For example, two protons combine to form a deuteron and a positron with release of 0.42 MeV energy :

$$ {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} \longrightarrow {}^{2}_{1}\text{H} + e^{+} + \nu + 0.42 \text{ MeV} $$

Similarly, two deuterons combine either to form the light isotope of helium and a neutron or a triton and a proton :

$$ {}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \longrightarrow {}^{3}_{2}\text{He} + {}^{1}_{0}n + 3.27 \text{ MeV} $$

$$ {}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \longrightarrow {}^{3}_{1}\text{H} + {}^{1}_{1}\text{H} + 4.03 \text{ MeV} $$

In all the above reactions, two positively charged particles combine to form a heavier nucleus. The fusing nuclei have to overcome very high electrostatic repulsion between them at extremely small distances. This repulsion prevents the two nuclei from getting close enough to be within the range of their attractive nuclear forces and thus 'fusing'. The height of this *coulomb barrier* depends on the charges and the radii of the two colliding nuclei. The height of potential barrier is higher for more highly charged nuclei.

To carry nuclear fusion in a bulk material, the temperature of the material has to be raised to 10^6 K, so that the colliding nuclei have enough energy due to their thermal motion and they can penetrate the coulomb barrier. This process is called ***thermonuclear fusion.***

13.41 NECESSARY CONDITIONS FOR NUCLEAR FUSION

46. *State the necessary conditions for nuclear fusion to occur.*

Necessary conditions for nuclear fusion. The fusion reactions take place under the conditions of extreme temperature and density due to the following reasons :

1. The ***high temperature*** is necessary for the light nuclei to have sufficient kinetic energy so that they can overcome their mutual coulombic repulsions and come closer than the range of nuclear force. That is why a fusion reaction is also called a **thermonuclear reaction.**

2. ***High density*** or pressure increases the frequency of collision of light nuclei and hence increases the rate of fusion.

These conditions exist in the interior of the sun where the temperature is about 2×10^6 K. Such conditions cannot be easily met in a laboratory.

13.42 FUSION AS A SOURCE OF ENERGY IN SUN AND STARS

47. *Explain the source of energy in sun and stars.*

Fusion as a source of energy in sun and stars. The sun has been radiating energy at the rate of 3.8×10^{26} Js^{-1} for several billion years without showing any sign of cooling off. A satisfactory explanation for this phenomenon was given by *H. Bethe* in 1939. Hydrogen nuclei, *i.e.*, protons are most abundant in the body of sun and stars. At extremely high temperatures which exist in interior of sun and stars, protons fuse together to form helium nuclei, liberating a huge amount of energy. This fusion takes place via *two* different cycles :

1. *Proton-proton cycle,* and

2. *Carbon-nitrogen cycle.*

1. **Proton-proton cycle**. The thermonuclear reactions in a proton-proton cycle take place in the following sequence :

$$ {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} \longrightarrow {}^{2}_{1}\text{H} + e^{+} + \nu + 0.42 \text{ MeV} $$

$$ e^{+} + e^{-} \longrightarrow \gamma + \gamma + 1.02 \text{ MeV} $$

$$ {}^{2}_{1}\text{H} + {}^{1}_{1}\text{H} \longrightarrow {}^{3}_{2}\text{He} + \gamma + 5.49 \text{ MeV} $$

$$ {}^{3}_{2}\text{He} + {}^{3}_{2}\text{He} \longrightarrow {}^{4}_{2}\text{He} + {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} + 12.86 \text{ MeV} $$

For the fourth reaction to occur, the first three reactions must occur twice so that two light helium nuclei (${}^{3}_{2}\text{He}$) may combine to form a normal helium nucleus (${}^{4}_{2}\text{He}$).

Then the net reaction will be

$$ 4\,{}^{1}_{1}\text{H} + 2e^{-} \longrightarrow {}^{4}_{2}\text{He} + 2\nu + 6\gamma + 26.7 \text{ MeV} $$

Thus four protons combine to form one helium nucleus with the liberation of 26.7 MeV of energy.

2. **Carbon-nitrogen cycle.** In this cycle, carbon nuclei successively absorb protons in a series of steps. Finally, they emit α-particles and again change into

carbon nuclei. The carbon-nitrogen cycle takes place in the following sequence :

$$^{12}_{6}C + ^{1}_{1}H \longrightarrow ^{13}_{7}N + \gamma + 1.93 \text{ MeV}$$

$$^{13}_{7}N \longrightarrow ^{13}_{6}C + e^{+} + \nu + 1.20 \text{ MeV}$$

$$^{13}_{6}C + ^{1}_{1}H \longrightarrow ^{14}_{7}N + \gamma + 7.6 \text{ MeV}$$

$$^{14}_{7}N + ^{1}_{1}H \longrightarrow ^{15}_{8}O + \gamma + 7.39 \text{ MeV}$$

$$^{15}_{8}O \longrightarrow ^{15}_{7}N + e^{+} + \nu + 1.71 \text{ MeV}$$

$$^{15}_{7}N + ^{1}_{1}H \longrightarrow ^{12}_{6}C + ^{4}_{2}He + 4.99 \text{ MeV}$$

The overall reaction may be represented as

$$^{1}_{1}H + ^{12}_{6}C \longrightarrow ^{4}_{2}He + ^{12}_{6}C + 2e^{+} + 2\nu + 3\gamma + 24.8 \text{ MeV}$$

Here again four protons combine to form a helium nucleus, gamma rays, and neutrinos and to liberate about 25 MeV of energy. Moreover, carbon is not consumed in the process but acts as a catalyst.

Temperature of the interior of the sun is 2×10^6 K. Both proton-proton and carbon-nitrogen cycles participate almost equally in the generation of energy in the sun. Stars, hotter than the sun, get their energy from the carbon-nitrogen cycle, while those cooler than the sun get their energy from the proton-proton cycle.

13.43 CONTROLLED THERMONUCLEAR REACTIONS

48. *What is controlled thermonuclear fusion ? State the essential requirements for a successful thermonuclear reactor.*

Controlled thermonuclear reactions. If the energy released in a thermonuclear reaction is controlled in such a manner that a limited amount of energy is produced continuously, it can be used for many useful purposes, particularly for generation of electrical power.

It is very difficult to set up a sustained and controllable source of fusion. The easiest thermo nuclear reaction that can be carried on earth is the fusion of two deuterons ($d-d$ reaction) or fusion of a deuteron with a triton ($d-t$ reaction).

$$^{2}_{1}H + ^{2}_{1}H \longrightarrow ^{3}_{2}He + ^{1}_{0}n + 3.2 \text{ MeV} \qquad (d-d)$$

$$^{2}_{1}H + ^{2}_{1}H \longrightarrow ^{3}_{1}H + ^{1}_{1}H + 4.0 \text{ MeV} \qquad (d-d)$$

$$^{2}_{1}H + ^{3}_{1}H \longrightarrow ^{4}_{2}He + ^{1}_{0}n + 17.6 \text{ MeV} \qquad (d-t)$$

Deuterium, the source of deuterons for the above reactions, has a very small isotopic abundance about 1 part in 7000, but it is available in plenty in sea-water.

Essential requirements for a thermonuclear reactor. The requirements for a successful thermonuclear reactor are as follows :

(*i*) ***High particle density.*** The number density of deuterium atoms must be sufficiently high so that the rate of $d-d$ collisions is high enough. At the high temperatures required for fusion, all the electrons get detached from the atoms. We have a mixture of electrons and deuterium nuclei (deuterons) moving at high speeds. The overall charge of the system is zero.

*Such a material consisting of moving charged particles with equal number of positive and negative charges is called **plasma**.*

(*ii*) ***High plasma temperature.*** The plasma must be hot enough so that the interacting particles may penetrate the coulomb barrier and hence undergo fusion. A plasma ion temperature of 35 keV, corresponding to 4×10^8 K has been attained in the laboratory.

(*iii*) ***Long confinement time.*** To ensure fusion of enough fuel, we need to confine hot plasma at sufficiently high temperature in a small volume for an extended time interval. No solid container can withstand such high temperatures. At present, two types of techniques are being used to confine hot plasma. In ***magnetic confinement***, also called ***tokamak design***, hot plasma is contained in a toroidal region by specially designed magnetic field. In ***inertial confinement***, lasers are used to confine hot plasma.

The energy produced by fusion is clean and is not accompanied by generation of any radioactive hazardous waste. Moreover, the fuel 'deuterium' used in fusion is available in unlimited quantity in sea-water. Efforts are being made to achieve controlled thermonuclear fusion in laboratory. Once this happens, fusion will become the ultimate source of unlimited and unpolluted energy.

Examples based on
(i) Q-value (ii) Nuclear Fission (iii) Nuclear Fusion

Formulae Used

1. Mass defect, Δm = Mass of reactant particles − Mass of product particles
2. Q-value = $(\Delta m)\, c^2$
3. Q-value is negative for endothermic reactions and positive for exothermic reactions.

Units Used

Mass defect Δm is in kg or in amu and Q-value in joule or in MeV. 1 amu = 931 MeV.

Example 31. *A neutron is absorbed by a ^6_3Li nucleus with subsequent emission of an alpha particle. Write the corresponding nuclear reaction. Calculate the energy released in this reaction.*

Given : $m(^6_3Li) = 6.015126$ *amu* ;

$m(^4_2He) = 4.0026044$ *amu.*

$(^1_0n) = 1.0086654$ *amu* ;

$m(^3_1H) = 3.016049$ *amu.*

[CBSE D 2000C, 02, OD 06]

Solution. $^6_3\text{Li} + ^1_0n \longrightarrow ^4_2\text{He} + ^3_1\text{H} + \text{Q}$

Initial masses	Final masses
$m(^6_3\text{Li}) = 6.015126$ amu	$m(^4_2\text{He}) = 4.0026044$ amu
$m(^1_0n) = 1.0086654$ amu	$m(^3_1\text{H}) = 3.016049$ amu
7.0237914 amu	7.0186534 amu

Mass defect,

$$\Delta m = 7.0237914 - 7.0186534$$
$$= 0.005138 \text{ amu}$$

Energy released,

$$Q = 0.005138 \times 931 \text{ MeV}$$
$$= \mathbf{4.78\ MeV.}$$

Example 32. *When a deuteron of mass 2.0141 amu and negligible kinetic energy is absorbed by a lithium (^6_3Li) nucleus of mass 6.0155 amu, the compound nucleus disintegrates spontaneously into two alpha particles, each of mass 4.0026 amu. Calculate the energy in joules carried by each alpha particle. (1 amu = 1.66×10^{-27} kg)* [CBSE OD 04]

Solution. $^6_3\text{Li} + ^2_1\text{H} \rightarrow ^4_2\text{He} + ^4_2\text{He} + Q$

$$m(^6_3\text{Li}) = 6.0155 \text{ amu}$$
$$m(^2_1\text{H}) = \mathbf{2.0141\ amu}$$

Total initial mass

$$= 8.0296 \text{ amu}$$

Total final mass

$$= 2\ m(^4_2\text{He}) = 2 \times 4.0026$$
$$= 8.0052 \text{ amu}$$

Mass defect,

$$\Delta m = 8.0296 - 8.0052$$
$$= 0.0244 \text{ amu}$$
$$= 0.0244 \times 1.66 \times 10^{-27} \text{kg}$$

Energy released,

$$Q = \Delta m \times c^2$$
$$= 0.0244 \times 1.66 \times 10^{-27} \times (3 \times 10^8)^2$$
$$= 3.645 \times 10^{-12} \text{ J}$$

Energy of each α-particle = $\mathbf{1.8225 \times 10^{-12}\ J.}$

Example 33. *The bombardment of lithium with protons gives rise to the following reaction :*

$$^7_3Li + ^1_1H \rightarrow ^4_2He + ^4_2He + Energy$$

The atomic masses of lithium, hydrogen and helium are 7.016 amu, 1.008 amu and 4.004 amu respectively. Find the initial energy of each α-particle (1 amu = 931 MeV).

[ISCE 95]

Solution. In terms of nuclear masses, the Q-value of the reaction is given by

$$Q = [m_N(^7_3\text{Li}) + m_N(^1_1\text{H}) - 2m_N(^4_2\text{He})]\, c^2$$

In terms of atomic masses, we can write

$$Q = [\{m(^7_3\text{Li}) - 3m_e\} + \{m(^1_1\text{H}) - m_e\} - 2\{m(^4_2\text{He}) - 2m_e\}]c^2$$
$$= [m(^7_3\text{Li}) + m(^1_1\text{H}) - 2m(^4_2\text{He})] \times c^2$$
$$= [7.016 + 1.008 - 2 \times 4.004] \times 931 \text{ MeV}$$
$$= 0.016 \times 931 = 14.896 \text{ MeV}$$

Energy of each α-particle $= \dfrac{14.896}{2} = \mathbf{7.448\ MeV.}$

Example 34. *Calculate the disintegration energy Q for the fission of $^{98}_{42}Mo$ into two equal fragments, $^{49}_{21}Sc$. If Q turns out to be positive, explain why this process does not occur spontaneously. Given that :*

$m(^{98}_{42}Mo) = 97.90541$ *amu*

$m(^{49}_{21}Sc) = 48.95002$ *amu*

$m_n = 1.00867$ *amu.*

[NCERT]

Solution. The fission of $^{98}_{42}$Mo can be represented as

$$^{98}_{42}\text{Mo} + ^1_0n \rightarrow 2\ ^{49}_{21}\text{Sc} + Q$$

The disintegration energy in the fission of $^{98}_{42}$Mo is given by

$$Q = [m(^{98}_{42}\text{Mo}) + m_n - 2m(^{49}_{21}\text{Sc})]\, c^2$$
$$= [97.90541 + 1.00867 - 2 \times 48.95002] \text{ amu} \times c^2$$
$$= [98.91408 - 97.90004] \text{ amu} \times \frac{931.5 \text{ MeV}}{\text{amu}}$$
$$= 1.01404 \times 931.5 = \mathbf{944.6\ MeV.}$$

Example 35. *If 200 MeV energy is released in the fission of a single nucleus of $^{235}_{92}U$, how many fissions must occur to produce a power of 1 kW ?* [Punjab 99, 03]

Solution. Let the number of fissions per second be n. Then,

Energy released per second

$$= n \times 200 \text{ MeV} = n \times 200 \times 1.6 \times 10^{-13} \text{ J}$$

Energy required per second

$$= \text{Power} \times \text{Time} = 1 \text{ kW} \times 1 \text{ s} = 1000 \text{ J}$$

$\therefore \quad n \times 200 \times 1.6 \times 10^{-13} = 1000$

or $\quad n = \dfrac{1000}{3.2 \times 10^{-11}} = \dfrac{10}{3.2} \times 10^{13} = \mathbf{3.125 \times 10^{13}}.$

Example 36. *Calculate the energy released by the fission of 1 g of $^{235}_{92}U$ in kWh. Energy per fission is 200 MeV.*

Solution. Number of atoms in 1 g of $^{235}_{92}U$

$$= \frac{\text{Avogadro's number}}{\text{Mass number}} = \frac{6.023 \times 10^{23}}{235}$$

Energy released per fission = 200 MeV

Energy released by fission of 1 g of $^{235}_{92}U$

$$= \frac{6.023 \times 10^{23}}{235} \times 200 \text{ MeV} = 5.126 \times 10^{23} \text{ MeV}$$

$$= 5.126 \times 10^{23} \times 1.6 \times 10^{-13} \text{ J}$$

$$= \frac{5.126 \times 1.6 \times 10^{10}}{3.6 \times 10^{6}} \text{ kWh} = \mathbf{2.278 \times 10^{4} \text{ kWh}}.$$

[$\because$ 1 kWh = 3.6×10^{6} J]

Example 37. *It is estimated that the atomic bomb exploded at Hiroshima released a total energy of 7.6×10^{13} J. If on the average 200 MeV energy was released by fission of one $^{235}_{92}U$ atom, calculate*

(*i*) *the number of uranium atoms fissioned*

(*ii*) *the mass of uranium used in the bomb.*

Solution. (*i*) Number of $^{235}_{92}U$ atoms fissioned,

$$n = \frac{\text{Total energy released}}{\text{Energy released per fission}}$$

$$= \frac{7.6 \times 10^{13}}{200 \times 1.6 \times 10^{-13}} = \mathbf{2.375 \times 10^{24}}.$$

(*ii*) Mass of uranium used

$$= \frac{\text{Mass number}}{\text{Avogadro's number}} \times n$$

$$= \frac{235 \times 2.375 \times 10^{24}}{6.023 \times 10^{23}} = \mathbf{926.66 \text{ g}}.$$

Example 38. *Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes :*

$$^{223}_{88}Ra \rightarrow \, ^{209}_{82}Pb + \, ^{14}_{6}C$$

$$^{223}_{88}Ra \rightarrow \, ^{219}_{86}Rn + \, ^{4}_{2}He$$

The Coulomb barrier height for alpha-particle emission is 30.0 MeV. What is the barrier height for $^{14}_{6}C$? The required data is

$$m(^{223}_{88}Ra) = 223.01850 \text{ amu}$$

$$m(^{209}_{82}Pb) = 208.98107 \text{ amu}$$

$$m(^{219}_{86}Rn) = 219.00948 \text{ amu}$$

$$m(^{14}_{6}C) = 14.00324 \text{ amu}$$

$$m(^{4}_{2}He) = 4.00260 \text{ amu}$$

Solution. The coulomb barrier height for α-decay is equal to the coulomb repulsion between the α-particle and the daughter nucleus when they are just touching each other.

The electrostatic potential energy between a particle with charge Z_1e and radius r_1, and a nucleus with charge Z_2e and radius r_2 is given by

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Z_1 e \cdot Z_2 e}{(r_1 + r_2)}$$

where $r_1 = r_0 A_1^{1/3}$ and $r_2 = r_0 A_2^{1/3}$.

For the emission of α-particle from $^{223}_{88}Ra$,

$$Z_1 = 2, \; A_1 = 4 \; ; \quad Z_2 = 86, \; A_2 = 219$$

$$\therefore U(\alpha) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2e \cdot 86e}{r_0(4^{1/3} + 219^{1/3})}$$

For the emission of $^{14}_{6}C$ from $^{223}_{88}Ra$,

$$Z_1 = 6, \; A_1 = 14 \; ; \quad Z_2 = 82, \; A_2 = 209$$

$$\therefore \quad U(^{12}_{6}C) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{6e \cdot 82e}{r_0(14^{1/3} + 209^{1/3})}$$

Hence

$$\frac{U(^{14}_{6}C)}{U(\alpha)} = \frac{6 \times 82}{2 \times 86} \times \frac{(219^{1/3} + 4^{1/3})}{(209^{1/3} + 14^{1/3})}$$

$$= \frac{6 \times 82}{2 \times 86} \times \frac{(6.3 + 1.5874)}{(5.935 + 2.410)}$$

$$= 2.86 \times \frac{7.8874}{8.345} = 2.703$$

or $\quad U(^{14}_{6}C) = 2.703 \, U(\alpha) = 2.703 \times 30 \text{ MeV}$

$$= 81.09 \approx \mathbf{81 \text{ MeV}}.$$

Example 39. *Two protons, each having a kinetic energy K, are fired at each other. What must K be if the particles are brought to rest by their mutual coulomb repulsion ? Assume a proton to be a sphere of radius $R = 1$ fm. Also estimate the temperature at which the protons can overcome this energy barrier.* [NCERT]

Solution. The initial mechanical energy, E_i of the two protons before collision is given by

$$E_i = 2K$$

When the protons stop, their entire energy is the electrostatic potential energy. It is given by

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e \times e}{(R + R)} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{2R}$$

By conservation of energy,

$$2K = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{2R}$$

$$\therefore \quad K = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{4R} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 1 \times 10^{-15}} \text{ J}$$

$[1 \text{ fm} = 10^{-15} \text{m}]$

$$= 5.75 \times 10^{-14} \text{ J} = \frac{5.75 \times 10^{-14}}{1.6 \times 10^{-16}} \text{ keV}$$

$$= 360 \text{ keV} \simeq \mathbf{400 \text{ keV}}.$$

This is approximately the coulomb barrier between two protons.

The temperature T at which protons in a proton gas would have enough energy to overcome the coulomb barrier between them is given by

$$\frac{3}{2} kT = K_{av}$$

or $$T = \frac{2K_{av}}{3k}$$

Here $K_{av} = 5.75 \times 10^{-14}$ J

Boltzmann constant,

$$k = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

$$\therefore \quad T = \frac{2 \times 5.75 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} \approx \mathbf{3 \times 10^9 \text{ K}}.$$

Example 40. *A star converts all its hydrogen to helium, achieving 100% helium composition. It then converts the helium to carbon via the reaction*

$$^4_2He + ^4_2He + ^4_2He \rightarrow ^{12}_6C + 7.27 \text{ MeV}$$

The mass of the star is 5.0×10^{32} kg, and it generates energy at the rate of 5×10^3 W. How long will it take to convert all the helium to carbon at this rate ? [NCERT]

Solution. Number of nuclei present in 4 g He

$$= 6 \times 10^{23}$$

Number of nuclei present in 5.0×10^{32} kg or 5.0×10^{35} g of helium

$$= \frac{6 \times 10^{23} \times 5.0 \times 10^{35}}{4} = 7.5 \times 10^{58}$$

Energy released in the fusion of 3 helium nuclei

$$= 7.27 \text{ MeV}$$

Energy released by the fusion of 7.5×10^{58} nuclei

$$= \frac{7.27 \times 7.5 \times 10^{58}}{3} \text{ MeV}$$

$$= 7.27 \times 2.5 \times 10^{58} \times 1.6 \times 10^{-13} \text{ J}$$

$$= 2.9 \times 10^{46} \text{ J}$$

Energy generated per second $= 5 \times 10^{30}$ J

Time taken to convert all helium nuclei into carbon

$$= \frac{2.9 \times 10^{46}}{5 \times 10^{30}} \text{ s} = 5.8 \times 10^{15} \text{ s}$$

$$= \frac{5.8 \times 10^{15}}{3.15 \times 10^7} \text{ years}$$

$$= \mathbf{1.8 \times 10^8 \text{ years}}.$$

Problems For Practice

1. How much mass has to be converted into energy to produce electric power 500 MW for one hour ? (Ans. 2×10^{-5} kg)

2. A slow neutron strikes a nucleus of $^{235}_{92}U$ splitting it into lighter nuclei of barium and krypton and releasing three neutrons. Write the corresponding nuclear reaction. Also calculate the energy released in this reaction.

   Given :

   $m(^{235}_{92}U) = 235.043933$ amu,

   $m(^1_0n) = 1.008665$ amu,

   $m(^{141}_{56}Ba) = 140.917700$ amu,

   $m(^{92}_{36}Kr) = 91.895400$ amu **[CBSE D 95]**

   (Ans. 198.77 MeV)

3. Find the Q-value for the nuclear reaction :

   $$^4_2He + ^{14}_7N \longrightarrow ^{17}_8O + ^1_1H$$

   $m(^4_2He) = 4.0039$ amu, $m(^{14}_7N) = 14.0075$ amu, $m(^{17}_8O) = 17.0045$ amu, $m(^1_1H) = 1.0082$ amu.

   (Ans. $\approx$ 1.21 MeV)

4. If 200 MeV energy is released in the fission of a single $^{235}_{92}U$ nucleus, how many fissions must occur to produce power of 1 kW ? **[Punjab 96C, 99]**

   (Ans. $3.125 \times 10^{13} \text{s}^{-1}$)

5. How much uranium $^{235}_{92}U$ should be consumed in a nuclear reactor for giving power of 10^6 watt, if the energy released per fission is 200 MeV ?

   (Ans. 0.044 g)

6. When an atom of $^{235}_{92}U$ undergoes fission, about 200 MeV energy is released. Suppose that a reactor using $^{235}_{92}U$ has an output of 700 MW and is 20% efficient. (*i*) How many uranium atoms does it consume in one day ? (*ii*) What mass of uranium does it consume each day ? (Ans. 9.5×10^{24}, 3.7 kg)

7. One MeV positron encounters 1 MeV electron travelling in opposite directions. What is the wavelength of photons produced ? Given rest mass energy of electron or positron = 0.512 MeV and $h = 6.62 \times 10^{-34}$ Js. (Ans. 8.2×10^{-13} m)

8. Consider one of the fission reactions of ^{235}U by thermal neutrons :

$$^{235}_{92}U + n \longrightarrow {}^{94}_{38}Sr + {}^{140}_{54}Ce + 2n$$

The fission fragments are, however, not stable. They undergo successive β-decays until $^{94}_{38}Sr$ becomes $^{94}_{40}Zr$ and $^{140}_{54}Xe$ becomes $^{140}_{58}Ce$. Estimate the total energy released in the process. Is all that energy available as kinetic energy of the fission products (Zr and Ce) ? You are given that

$m(^{235}U) = 235.0439$ amu, $m_n = 1.00866$ amu, $m(^{94}Zr) = 93.9065$ amu, $m(^{140}Ce) = 139.9055$ amu. **[NCERT]**

(Ans. 208 MeV)

9. Calculate the energy generated in kilowatt hours, when 100 g of $^{7}_{3}Li$ are converted into $^{4}_{2}He$ by proton bombardment. Given : mass of $^{7}_{3}Li$ atom = 7.0183 amu, mass of $^{4}_{2}He$ atom = 4.0040 amu, mass of $^{1}_{1}H$ atom = 1.0081 amu. (Ans. 6.5475×10^6 kWh)

10. In a star, three alpha particles join in a single reaction to form $^{12}_{6}C$ nucleus. Calculate the energy released in the reaction.

Given : $m(^{4}_{2}He) = 4.002604$ amu,

$m(^{12}_{6}C) = 12.000000$ amu. (Ans. 7.27 MeV)

11. Assuming that four hydrogen atoms combine to form a helium atom and two positrons, each of mass 0.000549 amu, calculate the energy released.

Given : $m(^{1}_{1}H) = 1.007825$ amu,

$m(^{4}_{2}He) = 4.002604$ amu. (Ans. 25.7 MeV)

12. The sun is believed to be getting its energy from the fusion of four protons to form a helium nucleus and a pair of positrons. Calculate the release of energy per fusion in MeV. Mass of a proton = 1.007825 amu ; mass of positron = 0.000549 amu ; mass of helium nucleus = 4.002603 amu ; 1 amu is equivalent to 931 MeV. (Ans. 25.70 MeV)

HINTS

1. Here $P = 500$ MW $= 5 \times 10^8$ W, $t = 1$ h $= 3600$ s

Energy produced,

$$E = P \times t = 5 \times 10^8 \times 3600 = 18 \times 10^{11} \text{ J}$$

As $E = mc^2$

$$\therefore \quad m = \frac{E}{c^2} = \frac{18 \times 10^{11}}{(3 \times 10^8)^2} = \frac{18 \times 10^{11}}{9 \times 10^{16}} = \mathbf{2 \times 10^{-5}\ kg}$$

2. $^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n + Q$

Initial masses :

$m(^{235}_{92}U) = 235.043933$ amu

$m(^{1}_{0}n) = 1.008665$ amu

$= 236.052598$ amu

Final masses :

$m(^{141}_{56}Ba) = 140.917700$ amu

$m(^{92}_{36}Kr) = 91.895400$ amu

$m(3\,^{0}_{1}n) = 3.025995$ amu

235.839095 amu

Mass defect = 236.052598 − 235.839095

= 0.213503 amu

Energy released, $Q = 0.213503 \times 931$ MeV

= **198.77 MeV.**

3. Use $Q = [m(^{4}_{2}He) + m(^{14}_{7}N) - m(^{17}_{8}O) - m(^{1}_{1}H)] \times c^2$

4. Total Energy released per second = 1 kJ = 10^{-3} J

Energy released per fission

$= 200 \text{ MeV} = 200 \times 1.6 \times 10^{-13} \text{ J} = 3.2 \times 10^{-11} \text{ J}$

No. of fissions per second

$$= \frac{10^3}{3.2 \times 10^{-11}} = \mathbf{3.125 \times 10^{13}}.$$

7. $^{0}_{-1}e + {}^{0}_{+1}e \longrightarrow 2\gamma$

Energy of each photon

$$= \frac{2(1 + 0.512)}{2} = 1.512 \text{ MeV}$$

$$= 1.512 \times 1.6 \times 10^{-13} \text{ J}$$

$$\lambda = \frac{hc}{E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1.512 \times 1.6 \times 10^{-13}} = \mathbf{8.2 \times 10^{-13}\ m.}$$

8. The net fission process is

$$^{235}_{92}U + {}^{0}_{1}n \longrightarrow {}^{94}_{40}Zr + {}^{140}_{54}Ce + 2\,{}^{1}_{0}n + 6e^- + Q$$

∴ Total energy released in the process is

$$Q = [m_N(^{235}_{92}U) - m_N(^{94}_{40}Zr) - m_N(^{140}_{40}Ce) - m_n - 6m_e]\,c^2$$

Here we have ignored the very small energy of thermal neutrons. We convert the nuclear masses into atomic masses :

$$Q = [\{m_N(^{235}_{92}U) + 92\,m_e\} - \{m_N(^{94}_{40}Zr) + 40m_e\}] - \{m_N(^{140}_{58}Ce) + 58m_e\} - m_n]\,c^2$$

$$= [m(^{235}_{92}U) - m(^{94}_{40}Zr) - m(^{140}_{58}Ce) - m_n]\,c^2$$

$$= [235.0439 - 93.9065 - 139.9055 - 1.00866] \times 931.5 \text{ MeV}$$

$$= (235.0439 - 234.82066) \times 931.5 \text{ MeV}$$

$$= 0.22324 \times 931.5 \text{ MeV} = 207.9 \text{ MeV} \simeq \mathbf{208\ MeV}$$

This entire energy is not available for fission products only. Part of it (about 20%) is carried by the neutrons produced in the fission process and also by the β-particles produced in the radioactive decay of the initial fission products.

9. The net reaction is

$$^{7}_{3}Li + {}^{1}_{1}H \rightarrow 2\,{}^{4}_{2}He + Q$$

$$\therefore Q = [m(^7_3\text{Li}) + m(^1_1\text{H}) - 2m(^4_2\text{He})] \times c^2$$
$$= [7.0183 + 1.0081 - 2 \times 4.0040] \text{ amu} \times \frac{931}{\text{amu}}$$
$$= 0.0184 \times 931 = 17.13 \text{ MeV}$$

Number of nuclei in 100 g of ^7_3Li

$$= \frac{6.02 \times 10^{23} \times 100}{7} = 8.6 \times 10^{24}$$

$\therefore$ Total energy released when 100 g of ^7Li is converted into ^4He nuclei

$$= 17.13 \times 8.6 \times 10^{24} \text{ MeV}$$
$$= 17.13 \times 8.6 \times 10^{24} \times 1.6 \times 10^{-13} \text{ J}$$
$$= \frac{17.13 \times 8.6 \times 1.6 \times 10^{11}}{3.6 \times 10^6} \text{ kWh}$$
$$= \mathbf{6.5475 \times 10^6 \text{ kWh}}.$$

10. $^4_2\text{He} + ^4_2\text{He} + ^4_2\text{He} \longrightarrow ^{12}_6\text{C} + Q$

Initial mass = Mass of three α-particles

$$= 3 \times 4.002604 \text{ amu} = 12.007812 \text{ amu}$$

Final mass = $m(^{12}_6\text{C}) = 12.000000$ amu

Mass defect, $\Delta m = 0.007812$ amu

Energy released,

$$Q = 0.007812 \times 931 \text{ MeV} = \mathbf{7.27 \text{ MeV}}.$$

11. $^1_1\text{H} + ^1_1\text{H} + ^1_1\text{H} + ^1_1\text{H} \longrightarrow ^4_2\text{He} + 2\,^0_1e + Q$

Initial mass = Mass of 4 hydrogen atoms

$$= 4 \times 1.007825 \text{ amu} = 4.031300 \text{ amu}$$

Final mass = $m(^4_2\text{He}) + 2\,m(^0_1e)$

$$= 4.002604 + 2 \times 0.000549$$
$$= 4.002604 + 0.001098 = 4.003702 \text{ amu}$$

Mass defect,

$$\Delta m = 4.031300 - 4.003702 = 0.027598 \text{ amu}$$

Energy released,

$$Q = 0.027598 \times 931 \text{ MeV} = \mathbf{25.7 \text{ MeV}}.$$

12. Proceed as in Problem 11 above.

13.44 NUCLEAR FISSION VERSUS NUCLEAR FUSION

49. *Give some important points of differences between nuclear fission and nuclear fusion.*

Nuclear fission	Nuclear fusion
1. Here a heavy nucleus when excited gets split up into two smaller nuclei of nearly comparable masses.	Here two lighter nuclei fuse together to form a heavier nucleus.
2. The conditions of high temperature and pressure are not necessary for its occurrence. It can be carried on the earth.	The conditions of extremely high pressure and temperature are necessary for its occurrence. So it cannot be easily carried in a laboratory.
3. Neutrons are the link particles of this process.	Protons are the link particles of this process.
4. It is a quick process.	It occurs in several steps. There is sufficient time gap between initial and final steps.
5. Here the energy available per nucleon is small, about 0.85 MeV.	Here the energy available per nucleon is large, about 6.75 MeV.
6. The energy obtained from a unit mass of a fissionable material is smaller than that obtained in case of fusion.	The energy obtained from a unit mass of a fusible material is large.
7. It produces very harmful radioactive wastes.	The products of fusion are harmless.
8. The stock of fissionable fusion is limited.	The fuel required for fusion is available in plenty.

13.45 NUCLEAR HOLOCAUST

50. *Briefly explain nuclear holocaust.*

Nuclear holocaust. This is a *nuclear era*. The discoveries of *nuclear fission* and *nuclear fusion* have made available to us vast sources of energy. For example, a single fission of uranium nucleus releases about 200 MeV of energy. Thus 50 kg of such nuclei will release 4×10^{15} J. This tremendous amount of energy is equivalent to about 20,000 tonnes of TNT, which is enough for a super-explosion. *The uncontrolled release of large energy is called atomic explosion.*

Seeing the enormous destructive power of atomic bombs, there was a mad race between the superpowers for acquiring and stockpiling more and more such bombs. For this a large number of nuclear tests were conducted and are being conducted by many countries. This created new problems like the disposal of radioactive wastes and environmental pollution caused by nuclear radiations. These high energy radiations have very harmful effects on living beings. The radioactive fall out from the increasing use of nuclear fission for generating controlled as well as uncontrolled energy and the actual use of atom bomb poses serious threat to mankind. Such a nuclear holocaust will not only destroy the life that exists now but it will render this planet unfit for life for all the times.

The radioactive wastes released from nuclear explosion will hang like a cloud in the earth's atmosphere and will absorb all the solar radiation. This will produce a long *nuclear winter* on the earth. So we should do our best to see that the possibility of a nuclear holocaust is ruled out. Should we use nuclear energy to improve quality of life on earth or use it to destroy the planet !

VERY SHORT ANSWER CONCEPTUAL PROBLEMS

Problem 1. **What will be the ratio of the radii of two nuclei of mass numbers A_1 and A_2 ?** [CBSE D 2000]

Solution. Ratio of the radii of two nuclei,

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

Problem 2. **What is the ratio of the nuclear densities of two nuclei having mass numbers in the ratio 1 : 4 ?**

Solution. As the nuclear density is independent of mass number, so the ratio of nuclear densities of the two given nuclei is 1 : 1.

Problem 3. **What does exactly make heavy nuclei unstable ?**

Solution. Heavy nuclei are unstable due to the presence of large repulsive forces exerted between a large number of protons.

Problem 4. **Why is the number of neutrons in heavier nuclei more than the number of protons ?**

Solution. In a heavier nucleus, the force of repulsion between protons is appreciable due to the presence of a large number of protons. In order to exert large attractive nuclear force and hence to maintain stability of the nucleus, more neutrons become necessary for a heavier nucleus.

Problem 5. **Why is the density of a nucleus much more than that of the atoms ?**

Solution. The nuclear size is of the order of 10^{-14} m and atomic size is of the order of 10^{-10} m. So atomic size is 10^4 times the nuclear size while atomic mass is slightly greater than nuclear mass due to the presence of electrons. Hence nuclear density is much greater than atomic density.

Problem 6. **The value of one unified atomic mass unit is 1.66×10^{-27} kg. Calculate the mass of one atom of ^{12}C in kilogram.** [ISCE 02]

Solution. 1 amu $= \frac{1}{12}$th mass of one atom of ^{12}C

$\therefore$ Mass of one atom of ^{12}C

$$= 12 \text{ amu} = 12 \times 1.66 \times 10^{-27} \text{ kg}$$

$$= \mathbf{1.992 \times 10^{-26} \text{ kg}}.$$

Problem 7. **What holds nucleons together in a nucleus ?** [Haryana 01]

Solution. The strong attractive nuclear force holds the nucleons together inside a nucleus which even overcomes the electrostatic repulsion between the protons.

Problem 8. **Why is the mass of a nucleus always less than the sum of the masses of its constituents, neutrons and protons ?** [CBSE D 06 ; OD 09]

Solution. When nucleons approach each other to form a nucleus, they strongly attract each other. Their potential energy decreases and becomes negative. It is this potential energy which holds the nucleons together in the nucleus. The decrease in potential energy results in the decrease in the mass of the nucleons inside the nucleus.

Problem 9. **Define mass defect of a nucleus. How is it related to the binding energy of the nucleus ?** [ISCE 97, 03]

Solution. The difference between the sum of the rest masses of the nucleons constituting a nucleus and the rest mass of the nucleus is called mass defect.

The binding energy of a nucleus is a measure of the energy equivalent to its mass defect

$$\text{B.E.} = \Delta m \times c^2$$

Problem 10. **The binding energies of deuteron (^2_1H) and α-particle (^4_2He) are 1.25 and 7.2 MeV/nucleon. Which nucleus is more stable ?** [CBSE D 96]

Solution. The α-particle (^4_2He) is more stable because of its higher B.E. per nucleon than that of deuteron (^2_1H).

Problem 11. **Electrons cannot be a part of a nucleus but protons can be a constituent part of it. Why ?**

Solution. This is because of the fact that the de-Broglie wavelength of electrons is larger than the size of the nucleus while that of protons is smaller than the size of the nucleus.

Problem 12. **Name two radioactive elements which are not found in observable quantities in nature. Why it is so ?** [CBSE D 95C]

Solution. Tritium and plutonium are two radioactive elements which are not found in observable quantities in nature. This is because their half-life is short as compared to the age of the universe.

Problem 13. **Why is it said that nuclear forces are saturated forces ?** [Punjab 04]

Solution. A nucleon interacts only with its nearest neighbouring nucleon. It does not interact with nucleons not in direct contact with it. That is why we say that nuclear forces show saturated effect. This is supported by the fact that binding energy per nucleon is same over a wide range of mass numbers.

Problem 14. **What do you mean by the charge independent character of nuclear forces ?**

Solution. The nuclear forces between two protons, between two neutrons or between a proton and a neutron are equally strong. So nuclear force does not depend on the charge of the particle. The electrostatic repulsion between two protons is overcome by the strong attractive nuclear force.

Problem 15. **Is free neutron a stable particle ? If not, what is its mode of decay ?** [CBSE D 92]

Solution. No, a free neutron is not a stable particle. It spontaneously decays into a proton, an electron and an antineutrino with a mean life of about 1000 s.

$$ {}_0^1n \rightarrow {}_1^1\text{H} + {}_{-1}^{\;0}e + \bar{\nu} $$

Problem 16. In the nuclear decay reaction

$$ {}_1^1\text{H} \rightarrow {}_0^1n + {}_Q^PX $$

find *P*, *Q* and hence identify *X*. [ISCE 95]

Solution. By conservation of mass, $P = 1 - 1 = 0$

By conservation of charge, $Q = 1 - 0 = 1$

$\therefore \quad X = {}_1^0e$ *i.e.*, *X* is a positron.

Problem 17. Can radioactivity be controlled ?

Solution. No, it cannot be controlled by changing (*i*) chemical conditions or (*ii*) the physical conditions like temperature, pressure, etc.

Problem 18. Natural radioactive nuclei are the nuclei of high mass number. Why ?

Solution. Heavy nuclei contain more neutrons than protons. They are unstable. In these nuclei, neutrons change into protons and vice versa. These processes involve emission of penetrating radiations.

Problem 19. Why do alpha particles have a high ionising power ? [CBSE F10]

Solution. Because of their large masses and large nuclear cross-section, α-particles have high ionising power.

Problem 20. What is the difference between an electron and a β-particle ? [Punjab 99C]

Solution. An electron and a β-particle are essentially the same. An electron of nuclear origin is called a β-particle.

Problem 21. A nucleus contains no electrons, yet it ejects them. How ?

Or

How are β-rays emitted from a nucleus, when it does not contain electrons ? [Punjab 03]

Solution. A neutron of a nucleus decays into a proton, an electron and an antineutrino. It is this electron which is emitted as β-particle.

$$ {}_0^1n \longrightarrow {}_1^1p + {}_{-1}^{\;0}e + \bar{\nu} $$

Problem 22. Why is the energy distribution of beta rays continuous ? [CBSE OD 98C]

Or

Why do all the electrons emitted during beta decay not have the same energy ? [CBSE OD 96]

Solution. In a β-decay, particles like antineutrinos are also emitted alongwith electrons. The available energy is shared by electrons and antineutrinos in all proportions. Electron's energy is no longer fixed. Hence energy distribution of β-rays is continuous.

Problem 23. When does a nucleus emit a γ-ray photon ?

Solution. After losing an α-or β-particle, a daughter nucleus is left in the excited state. It comes to its ground state by emitting one or more γ-ray photons.

Problem 24. Why are γ-rays also called electromagnetic waves ?

Solution. γ-rays consist of photons of short wavelength which travel with the speed of e.m. waves and show properties similar to e.m. waves. So γ-rays are called e.m. waves.

Problem 25. Why are γ-rays not deflected by electric and magnetic fields ?

Solution. γ-rays consist of neutral photons which cannot be deflected by electric and magnetic fields.

Problem 26. Why are α-particles emitted rather than protons or ${}_2^3$He nuclei ?

Solution. This is because α-particles have a high value of binding energy. With the emission of α-particle, the binding per nucleon of the residual nucleus increases appreciably. In contrast, the emission of a proton or ${}_2^3$He nuclei may not be energetically favourable.

Problem 27. Why it is not possible to define total life of a radioactive substance ?

Solution. Radioactivity is a spontaneous process which occurs by chance only. Since a nucleus can have any value of total life between zero and infinity, so it is not possible to define the total life of a radioactive substance.

Problem 28. A radioactive sample having *N* nuclei has activity *R*. Write down an expression for its half-life in terms of *R* and *N*. [ISCE 97]

Solution. Activity, $R = N\lambda$

$\therefore$ Disintegration constant, $\lambda = \dfrac{R}{N}$

Half-life, $T_{1/2} = \dfrac{0.693}{\lambda} = \dfrac{0.693\,N}{R}$.

Problem 29. The radioactivity of the sample is R_1 at time t_1 and R_2 at time t_2. The mean life of the sample is τ. What is the number of nuclei that have disintegrated in the time interval $(t_1 - t_2)$?

Solution. Let N_1 and N_2 be the number of undecayed nuclei present at times t_1 and t_2 respectively. Then

$$ R_1 = \left|\frac{dN_1}{dt}\right| = \lambda\, N_1 \text{ and } R_2 = \left|\frac{dN_2}{dt}\right| = \lambda\, N_2 $$

$$ \therefore \quad R_1 - R_2 = \lambda(N_1 - N_2) $$

or $$ N_1 - N_2 = \frac{R_1 - R_2}{\lambda} $$

Clearly, $(N_1 - N_2)$ is the number of nuclei that have disintegrated in time interval $(t_1 - t_2)$.

Problem 30. Show that the decay rate 'R' of a sample of a radionuclide is related to the number of radioactive nuclei 'N' at the same instant by the expression $R = \lambda N$. [CBSE D 05]

Solution. By radioactive decay law, $N = N_0 e^{-\lambda t}$

Rate of decay,

$$R = -\frac{dN}{dt} = -\frac{d}{dt}(N_0 e^{-\lambda t}) = \lambda N_0 e^{-\lambda t} \quad \text{or} \quad R = \lambda N.$$

Problem 31. Tritium has a half-life of 12.5 years against beta decay. What fraction of a sample of pure tritium will remain undecayed after 25 years? [NCERT ; Himachal 93]

Solution. From the definition of half-life, we can say that

(*i*) $\frac{1}{2}$ of the initial tritium nuclei remain undecayed after the first 12.5 years,

(*ii*) $\frac{1}{2}$ of the remaining nuclei will decay in the next 12.5 years, and hence

(*iii*) $\frac{1}{4}$ of the initial tritium nuclei will remain undecayed after 25 years.

Problem 32. What percentage of a given mass of a radioactive substance will be left undecayed after five half-life periods? [CBSE F 94]

Solution. Here, number of half-life periods $(n) = 5$

$$\therefore \quad \frac{N}{N_0} = \frac{1}{2^n} = \frac{1}{2^5} = \frac{1}{32} = \mathbf{3.125\%}.$$

Problem 33. A radioactive substance decays to 1/32 th of its initial activity in 25 days. Calculate its half life. [Himachal 03]

Solution. Here $R = R_0/32$

But $\quad R = R_0 \left(\frac{1}{2}\right)^n$

$$\therefore \quad \frac{R_0}{32} = R_0 \left(\frac{1}{2}\right)^n \quad \text{or} \quad \left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^n$$

or $\quad n = 5$

Half-life period

$$= \frac{\text{Time of disintegration}}{\text{No. of half-lives}} = \frac{25}{5} = \mathbf{5\ days}$$

Problem 34. Why is it found experimentally difficult to detect neutrinos in nuclear β-decay? [CBSE OD 14, 15C ; F 15]

Solution. Neutrinos are massless and chargeless particles. They are difficult to detect because

(*i*) they interact very weakly with other particles.

(*ii*) they can penetrate large thicknesses of matter without any interaction.

Problem 35. In a radioactive decay, a nucleus emits an α-particle and two β-particles successively. Show that the final nucleus is an isotope of the original nucleus.

Solution. The entire decay process may be represented as

$${}^{A}_{Z}X \xrightarrow{{}^{4}_{2}\text{He}} {}^{A-4}_{Z-2}Y \xrightarrow{{}^{0}_{-1}e} {}^{A-4}_{Z-1}Z \xrightarrow{{}^{0}_{-1}e} {}^{A-4}_{Z}X'$$

As both X' and X have the same atomic number Z, so the final nucleus is an isotope of the original nucleus.

Problem 36. Why is neutron so effective as bombarding particle?

Solution. A neutron carries no charge. It easily penetrates even a heavy nucleus without being repelled or attracted by nucleus and electrons. So it serves as an ideal projectile for starting a nuclear reaction.

Problem 37. Why is ${}^{238}_{92}U$ not suitable for chain reaction? [Himachal 95, 01]

Solution. Only fast moving neutrons of 12 MeV can cause fission of ${}^{238}_{92}U$ nuclei. But such neutrons have less chances of interaction. They escape the fissionable material without causing fission.

Problem 38. Why are the control rods made of cadmium?

Solution. Cadmium has high cross-section for the absorption of neutrons.

Problem 39. What is meant by multiplication factor (k) of a fissionable material? For what value of k, a chain reaction will grow?

Solution. The multiplication factor of a fissionable material is defined as the ratio of the number of neutrons present at the beginning of a particular generation to the number of neutrons present at the beginning of previous generation. A chain reaction grows only when $k > 1$.

Problem 40. Why is nuclear fusion difficult to carry out? [CBSE OD 03C]

Or

Why is nuclear fusion not possible in a laboratory? [Punjab 09]

Solution. Nuclear fusion requires very high temperature of $10^6 - 10^7$ K. This temperature is attained by causing explosion due to the fission process. Moreover, no solid container can withstand such a high temperature.

Problem 41. Why do lighter nuclei tend to fuse together?

Solution. When lighter nuclei fuse together, they form heavier nuclei having greater binding energy per nucleon and they tend to attain a stable structure.

Problem 42. Why are fusion reactions also known as thermonuclear reactions?

Solution. To overcome coulomb repulsion, the fusing nuclei are given enough thermal energy by raising their temperature to $10^6 - 10^7$ K. That is why nuclear fusion is also called thermonuclear fusion.

Problem 43. **Why should nuclear fission precede a nuclear fusion ?**

Solution. Nuclear fusion occurs at very high temperature of $10^6 - 10^7$ K. Such a high temperature can be obtained by atomic explosion involving nuclear fission. Once the fusion process is initiated, the energy released is enough to sustain this process.

Problem 44. **A fusion reaction is more energetic than a fission reaction. Comment.** [Punjab 10, 11]

Solution. The energy released per unit mass of the fuel in a fusion reaction is much more than the energy released per unit mass of the fuel in fission reaction.

Problem 45. **The sun is constantly losing mass due to thermonuclear fusion. Comment.** [Punjab 10]

Solution. In each fusion reaction, a small mass of the sun changes into thermal energy. So sun is constantly losing mass due to thermonuclear fusion.

Problem 46. **Some scientists have predicted that a global nuclear war on earth would be followed by 'nuclear winter'. What would cause 'nuclear winter' ?** [CBSE OD 95]

Solution. The clouds produced by a global nuclear war would perhaps cover major parts of the sky preventing solar light from reaching many parts of the earth. This would cause a 'nuclear winter'.

Problem 47. **In the nuclear reaction**

$$_1^1\text{H} \rightarrow {}_0^1n + {}_Q^PX$$

Find P, Q and hence identify X. [ISCE 95]

Solution. Using the laws of conservation of mass and charge, we get

$$P + 1 = 1 \quad \text{or} \quad P = 0$$
$$Q + 0 = 1 \quad \text{or} \quad Q = 1$$

Thus ${}_Q^PX$ is a positron $({}_{+1}^{0}e)$.

Problem 48. **When ${}_3^7$Li is bombarded with a certain particle, two alpha particles are produced. Identify the bombarding particle.** [ISCE 98]

Solution. Let ${}_Z^AP$ be the bombarding particle.

Then

$$_3^7\text{Li} + {}_Z^AP \rightarrow {}_2^4\text{He} + {}_2^4\text{He}$$

Using the laws of conservation of mass and energy, we get

$$A + 7 = 4 + 4 \quad \text{or} \quad A = 1$$
$$Z + 3 = 2 + 2 \quad \text{or} \quad Z = 1$$

Thus, the bombarding particle is a proton $({}_1^1\text{H})$.

Problem 49. **State the reason, why heavy water is generally used as a moderator in a nuclear reactor.** [CBSE D 08]

Solution. Heavy water contains protons (of mass nearly that of neutrons). Fast moving neutrons undergo elastic collisions with these slow moving neutrons and thus get slowed down. Hence heavy water can be used as a moderator. Also, heavy water has negligible cross-section for neutron absorption.

SHORT ANSWER CONCEPTUAL PROBLEMS

Problem 1. **Calculate the energy equivalent (in MeV) of one twelfth of the mass of one atom of carbon 12.** [ISCE 01]

Solution. Mass of one atom of ^{12}C

$= 12 \text{ amu} = 12 \times 1.66 \times 10^{-27}$ kg

Energy equivalent of this mass is

$$E = mc^2 = 12 \times 1.66 \times 10^{-27} \times (3 \times 10^8)^2 \text{ J}$$
$$= \frac{12 \times 1.66 \times 9 \times 10^{-11}}{1.6 \times 10^{-13}} \text{ MeV}$$
$$= \mathbf{11205 \text{ MeV}}.$$

Problem 2. **The atomic mass of an element is the weighted average of the atomic masses of different isotopes of the element. This explains, why atomic masses of many elements show large departures from integer values. However, even if we consider masses of individual isotopes, they are not strictly integer multiples of the mass of a hydrogen atom. How do you account for this fact ?**

Solution. This is because of following reasons :

1. The mass of proton and neutron are not identical.
2. The atomic mass also includes mass of electrons and an atom of mass number A has Z electrons and not A electrons.
3. The mass of nucleus is slightly less than the mass of the constituent nucleons.

Problem 3. **The isotopes ${}_8^{16}$O has 8 protons, 8 neutrons and 8 electrons, while ${}_4^8$Be has 4 protons, 4 neutrons and 4 electrons. Yet the ratio of their atomic masses is not exactly 2. Why ?**

Solution. This is because of the fact that the mass of a nucleus is slightly less than the mass of the constituent nucleons. This decrease in mass is called mass defect. Since, the mass defect in case of ${}_8^{16}$O is not exactly twice of the mass defect in case of ${}_4^8$Be, the ratio of the atomic masses is not exactly 2.

Problem 4. **You are given two nuclides ^7_3X and ^4_3Y.**

(i) Are they the isotopes of the same element ? Why ?

(ii) Which one of the two is likely to be more stable ? Give reason. [CBSE OD 2000, F 06]

Solution. (i) Yes, they are the isotopes of the same element because they have same atomic number ($Z = 3$).

(ii) The isotope ^7_3X has 3 protons and 4 neutrons while the isotope ^4_3Y has 3 protons and 1 neutron. Due to the presence of a greater number of neutrons in ^7_3X, the strong attractive nuclear force dominates over the electrostatic repulsion between the protons. So ^7_3X is more stable than ^4_3Y.

Problem 5. **Group the following six nuclides into three pairs of (i) isotones, (ii) isotopes and (iii) isobars :**

$$^{12}_{6}C,\ ^{3}_{2}He,\ ^{198}_{80}Hg,\ ^{3}_{1}H,\ ^{197}_{79}Au,\ ^{14}_{6}C$$

How does the size of a nucleus depend on its mass number ? Hence explain why the density of nuclear matter should be independent of the size of the nucleus. [CBSE OD 04C ; D 13]

Solution. (i) Isotones : $^{198}_{80}Hg$ and $^{197}_{79}Au$. [*Same* $(A-Z)$]

(ii) Isotopes : $^{12}_{6}C$ and $^{14}_{6}C$. [*Same Z but different A*]

(iii) Isobars : $^{3}_{2}He$ and $^{3}_{1}H$. [*Same A but different Z*]

The effective radius of a nucleus is related to its mass number A as $r = r_0\ A^{1/3}$, r_0 is a constant.

Volume of the nucleus

$$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (r_0\ A^{1/3})^3 = \frac{4}{3}\pi r_0^3\ A$$

If m is the average mass of a nucleon, then mass of the nucleus $= mA$

$$\text{Nuclear density} = \frac{\text{Mass}}{\text{Volume}} = \frac{mA}{\frac{4}{3}\pi r_0^3\ A} = \frac{3\,m}{4\pi r_0^3}$$

Clearly, nuclear density is independent of the size of the nucleus.

Problem 6. **What is meant by binding energy per nucleon ? The binding energies of deuteron (^2_1H) and α-particle (^4_2He) are 1.25 and 7.2 MeV/nucleon respectively. Which nucleus is more stable ?** [CBSE OD 2000C]

Solution. Binding energy per nucleon is the average energy required to remove a nucleon from a nucleus. As the binding energy per nucleon of ^4_2He is greater than that of ^2_1H, so ^4_2He nucleus is more stable than ^2_1H nucleus.

Problem 7. **A heavy nucleus X of mass number $A = 240$ and binding energy per nucleon 7.6 MeV is split into two nearly equal fragments Y and Z of mass numbers $A_1 = 110$ and $A_2 = 130$. The binding energy of each one of these nuclei is 8.5 MeV per nucleon. Calculate the total binding energy of each of the nuclei X, Y and Z and hence the energy Q released per fission in MeV.** [ISCE 03 ; CBSE D 10]

Solution. *For nucleus* X : $A = 240$

B.E. per nucleon = 7.6 MeV

Total B.E. of X $= 240 \times 7.6 = 1824$ MeV

For nucleus Y : $A_1 = 110$

B.E. per nucleon = 8.5 MeV

Total B.E. of Y $= 110 \times 8.5 = 935$ MeV

For nucleus Z : $A_2 = 130$

B.E. per nucleon = 8.5 MeV

Total B.E. of Z $= 130 \times 8.5 = 1105$ MeV

Energy released per fission,

$$Q = \text{B.E. of } Y + \text{B.E. of } Z - \text{B.E. of } X$$
$$= 935 + 1105 - 1824 = 2040 - 1824 = \mathbf{216\ MeV}.$$

Problem 8. **Write down the value of charge and mass of an α-particle. Express charge in terms of electronic charge and the mass in terms of the mass of a proton. Can an α-particle be compared with a helium atom ?**

Solution. Charge on α-particle $= 2e$

$= 2 \times$ Charge on an electron $= 3.2 \times 10^{-19}$ C.

Mass of α-particle

$= 4m_p = 4 \times$ Mass of proton $= 4.00260$ amu.

An α-particle is a doubly ionised helium atom i.e., it is a helium nucleus.

Problem 9. **Give the nature of α-, β- and γ- radiations.**

Solution. α-particles are helium nuclei which carry positive charge.

β-particles are fast moving electrons which carry negative charge.

γ-rays are neutral photons of electromagnetic nature.

Problem 10. **The half life period of a radioactive element A is the same as the mean life of another radioactive element B. Initially both of them have the same number of atoms. The radioactive element B decays faster than A. Explain, why.** [IIT 99 ; Punjab 01]

Solution. If λ and λ' are the decay constants of the elements A and B respectively, then

$$T_{1/2}(A) = \tau(B)$$

or

$$\frac{0.693}{\lambda} = \frac{1}{\lambda'} \quad \text{or} \quad \frac{\lambda}{\lambda'} = 0.693$$

If both the samples have N atoms each initially, then the ratio of their rates of disintegration will be

$$\frac{R}{R'} = \frac{\lambda N}{\lambda' N} = \frac{\lambda}{\lambda'} = 0.693$$

Clearly, $R' > R$ *i.e.*, the element B disintegrates faster than A.

Problem 11. Explain how radioactive nuclei can emit β-particles even though atomic nuclei do not contain these particles. Hence explain why the mass number of a radioactive nuclide does not change during β-decay. [CBSE D 04C]

Solution. A neutron from the nucleus decays to emit a proton, an electron and an antineutrino. The electron is emitted in the form of a β-particle.

$$_0^1 n \longrightarrow {}_1^1 p + {}_{-1}^{0} e + \bar{\nu}$$

During β-decay, a neutron simply changes into a proton *i.e.,* only the character of nucleon changes. Hence the mass number of a radioactive nuclide does not change during a β-decay.

Problem 12. A nucleus $_n^m X$ emits one alpha particle and one beta particle. Find the mass number and atomic number of the product nucleus. [CBSE D 03]

Solution. Using the law of conservation of mass and the law of conservation of charge, we rewrite the decays as

$$_n^m X \xrightarrow[\alpha\text{-decay}]{_2^4 He} {}_{n-2}^{m-4} Y \xrightarrow[\beta\text{-decay}]{_{-1}^{0} e} {}_{n-1}^{m-4} Z$$

Mass number of the product nucleus = $m - 4$

Atomic number of the product nucleus = $n - 1$.

Problem 13. With the help of an example, explain, how the neutron to proton ratio changes during the alpha decay of a nucleus. [CBSE D 04, 06]

Solution. Consider the α-decay :

$$_{92}^{238} U \xrightarrow{_2^4 He} {}_{90}^{234} Th$$

Neutron-proton ratio before α-decay

$$= \frac{238 - 92}{92} = \frac{146}{92}$$

Neutron-proton ratio after α-decay

$$= \frac{234 - 90}{90} = \frac{144}{90}$$

Clearly, $\frac{144}{90} > \frac{146}{92}$

Thus the neutron-proton ratio increases during an α-decay.

Problem 14. Explain with an example, whether the neutron-proton ratio in a nucleus increases or decreases due to beta (β) decay. [CBSE D 06 ; OD 03]

Solution. In a β-decay, a neutron gets converted into a proton. So the neutron-proton ratio decreases.

Consider the β-decay :

$$_{83}^{210} Bi \longrightarrow {}_{84}^{210} Po + {}_{-1}^{0} e$$

Neutron-proton ratio before β-decay

$$= \frac{210 - 83}{83} = \frac{127}{83}$$

Neutron-proton ratio after β-decay

$$= \frac{210 - 84}{84} = \frac{126}{84}$$

As $\frac{126}{84} < \frac{127}{83}$.

Thus the neutron-proton ratio decreases in a β-decay.

Problem 15. Explain why the β-decay of a free proton is not possible but that of a proton bound in the nucleus is possible ? [CBSE Sample Paper 90]

Solution. A free neutron has rest mass energy greater than that of a proton. Thus β-decay

$$_0^1 n \longrightarrow {}_1^1 p + {}_{-1}^{0} e + \bar{\nu}$$

is energetically allowed, but the β-decay of a free proton into a neutron :

$$_1^1 p \longrightarrow {}_0^1 n + {}_1^0 e + \nu$$

is not allowed energetically.

Inside a nucleus, individual neutrons and protons are not free. Thus the β^+-decay of a proton is possible when the proton is bound in a nucleus. The energy needed for the decay can come from the appropriate difference in binding energies of a proton and a neutron in the nucleus.

Problem 16. Explain what is meant by radioactive decay. A radioactive nucleus is represented by the symbol $_b^a V$. How is the new nucleus represented after the emission of (*i*) an alpha particle, (*ii*) a beta particle and (*iii*) a gamma ray ? The activity of a source undergoing a single type of decay is R_0 at time $t = 0$. Obtain an expression in terms of the half-life. $T_{1/2}$ for the activity R at any subsequent time t. [CBSE OD 01C]

Solution. Radioactive decay is the spontaneous disintegration of the nucleus of an atom with the emission of one or more penetrating radiations like α, β and γ-rays.

(*i*) α-decay : $_b^a V \rightarrow {}_{b-2}^{a-4} X + {}_2^4 He + Q$

(*ii*) β-decay : $_b^a V \rightarrow {}_{b+1}^{a} X + {}_{-1}^{0} e + \bar{\nu} + Q$

(*iii*) γ-decay : $_b^a V \rightarrow {}_b^a V + \gamma$
(Excited state → Ground state)

Activity, $R = -\frac{dN}{dt}$

According to radioactive decay law, $-\frac{dN}{dt} = \lambda N$

$\therefore \quad R = \lambda N$

As $\quad N = N_0 e^{-\lambda t}$

$\therefore \quad R = \lambda N e^{-\lambda t}$ or $R = R_0 e^{-\lambda t}$

where $\lambda N_0 = R_0$ = activity of the sample at $t = 0$.

Also $\quad \lambda = \frac{0.693}{T_{1/2}}$

$$\therefore \quad R = R_0 e^{-\frac{0.693 t}{T_{1/2}}}.$$

Problem 17. Define decay constant of a radioactive sample. Which of the following radiations, α-rays, β-rays, γ-rays

(i) are similar to X-rays ?

(ii) are easily absorbed by matter ?

(iii) travel with greatest speed ?

(iv) are similar in nature to cathode rays ?

[CBSE F 95 ; OD 01]

Solution. Decay constant is the reciprocal of the time interval in which the number of active nuclei in a radioactive sample reduces to $1/e$ times of its initial value.

(i) γ-rays are similar to X-rays.

(ii) α-rays are easily absorbed by matter.

(iii) γ-rays travel with greatest speed.

(iv) β-rays are similar to cathode rays.

Problem 18. Write the nuclear reactions for the following :

(i) α-decay of $^{204}_{84}Po$ (ii) β^--decay of $^{32}_{15}P$

(iii) β^+-decay of $^{11}_{6}C$

[NCERT ; CBSE D 05C ; OD 12]

Solution. (i) $^{204}_{84}Po \rightarrow ^{200}_{82}Pb + ^{4}_{2}He$

(ii) $^{32}_{15}P \rightarrow ^{32}_{16}S + ^{0}_{-1}e + \bar{\nu}$

(iii) $^{11}_{6}C \rightarrow ^{11}_{5}B + ^{0}_{+1}e + \nu$.

Problem 19. A radioactive nucleus 'A' decays as given below :

$$A \xrightarrow{^{0}_{-1}e} A_1 \xrightarrow{\alpha} A_2$$

If the mass number and atomic number of A_1 are 180 and 73 respectively, find the mass number and atomic number of A and A_2.

[CBSE OD 03C]

Solution. Using displacement laws of radioactive transformation, we can represent the successive decays as follows :

$$^{180}_{72}A \xrightarrow{^{0}_{-1}e} {}^{180}_{73}A_1 \xrightarrow{^{4}_{2}He} {}^{176}_{71}A_2$$

Mass number of $A = 180$

Atomic number of $A = 72$

Mass number of $A_2 = 176$

Atomic number of $A_2 = 71$.

Problem 20. The sequence of stepwise decays of a radioactive nucleus is

$$D \xrightarrow{\alpha} D_1 \xrightarrow{\beta^-} D_2 \xrightarrow{\alpha} D_3$$

If the nucleon number and atomic number of D_2 are 176 and 71 respectively, what are the corresponding values of D and D_3 ? Justify your answer in each case.

[CBSE OD 05C]

Solution. The decays can be represented as follows :

$$^{180}_{72}D \xrightarrow{^{4}_{2}He} {}^{176}_{70}D_1 \xrightarrow{^{0}_{-1}e} {}^{176}_{71}D_2 \xrightarrow{^{4}_{2}He} {}^{172}_{69}D_3$$

Nucleon number of $D = 180$

Atomic number of $D = 72$

Nucleon number of $D_3 = 172$

Atomic number of $D_3 = 69$.

Problem 21. (a) If the α-decay of ^{238}U is energetically allowed (i.e., the decay products have a total mass less than the mass of ^{238}U), what prevents ^{238}U from decaying all at once ? Why is its half life so large ?

(b) The α-particle faces a Coulomb barrier. A neutron being uncharged faces no such barrier. Why does the nucleus $^{238}_{92}U$ not decay spontaneously by emitting a neutron ?

[NCERT]

Solution. (a) The α-decay is caused by the quantum mechanical tunnelling of the nucleus by the α-particles. The rate of tunnelling is determined by the height of the nuclear potential barrier, its width etc.

(b) The possible decay would be

$$^{238}_{92}U \longrightarrow ^{237}_{92}U + ^{1}_{0}n$$

$$m(^{237}_{92}U) = 237.04874 \text{ amu}$$

$$m_n = \underline{1.00867 \text{ amu}}$$

$$\text{Total mass} \qquad = 238.05741 \text{ amu}$$

$$m(^{238}_{92}U) = 238.05081 \text{ amu}$$

i.e., $$m(^{237}_{92}U) + m_n > m(^{238}_{92}U)$$

Thus the spontaneous decay of $^{238}_{92}U$ is not energetically possible, *i.e.*, it will not emit a neutron spontaneously. Rather, energy would be needed to separate a neutron from $^{238}_{92}U$.

Problem 22. (a) The observed decay products of a free neutron are a proton and an electron. The emitted electrons are found to have a continuous distribution of kinetic energy with a maximum of $(m_n - m_p - m_e)c^2$. Explain clearly why the presence of a continuous distribution of energy is a pointer to the existence of other unobserved product(s) in the decay.

(b) If a neutron is unstable with a half life of about 1000 s, why don't all the neutrons of a nucleus decay eventually into protons ? How can a nucleus of Z protons and (A − Z) neutrons ever remain stable if the neutrons themselves are unstable ?

Solution. (a) Consider the decay of a free neutron at rest.

$$^{1}_{0}n \longrightarrow ^{1}_{1}p + ^{0}_{-1}e$$

By momentum conservation, if the momentum of electron is p_e, then the momentum of proton is $-p_e$. By energy conservation,

$$m_n c^2 = [p_e^2 c^2 + m_e^2 c^4]^{1/2} + [p_e^2 c^2 + m_p^2 c^4]^{1/2}$$

Thus a definite momentum p_e is given to the electron. This means the energy of the electron in the above decay is fixed. An electron in the above decay cannot have a continuous distribution of energies. The presence of an additional particle, however, allows this possibility. The available energy can now be shared by the electron and the third particle, and the electron's energy is no longer fixed. This led Pauli to postulate the existence of a new particle till then unobserved. We now know that the correct equation for β-decay is : ${}_0^1n \longrightarrow {}_1^1p + {}_{-1}^{0}e + \bar{\nu}$

where the particle denoted by $\bar{\nu}$ is called antineutrino. It is a neutral particle of negligibly small rest mass and intrinsic spin 1/2.

(*b*) A free neutron has rest mass greater than that of a proton. Thus β^--decay is energetically allowed, but the β^+-decay of a proton into a neutron is not allowed. In a nucleus, individual neutrons and protons are not free. Thus the β^+-decay of a proton ($p \longrightarrow n + e^+ + \nu_e$) *is possible when the proton is bound in a nucleus.* The energy needed for the decay comes from the difference in binding energies of a proton and a neutron in the nucleus. In a stable nucleus with Z protons and $(A - Z)$ neutrons, the two reciprocal processes (neutron decay and proton decay) are in dynamic equilibrium.

Problem 23. **If both the number of protons and neutrons in a nuclear reaction is conserved, in what way is mass converted into energy (or vice versa) ? Explain giving one example.** Or [CBSE D 15C]

If the total number of neutrons and protons in a nuclear reaction is conserved, how then is the energy absorbed or evolved in the reaction ? Explain. [CBSE D 06]

Solution. As the number of nucleons is conserved, the total rest mass of protons and neutrons on either side of the reaction remains same. But the binding energies of nuclei on the two sides of the reaction are different. It is this difference in the biding energies that appears as the energy released in the nuclear reaction.

Example. ${}_1^2H + {}_1^2H \rightarrow {}_2^3He + {}_0^1n +$ energy.

Problem 24. **Give the mass number and atomic number of elements on the right-hand side of the decay process.**

$${}_{86}^{220}\text{Ru} \rightarrow \text{Po} + \text{He}$$

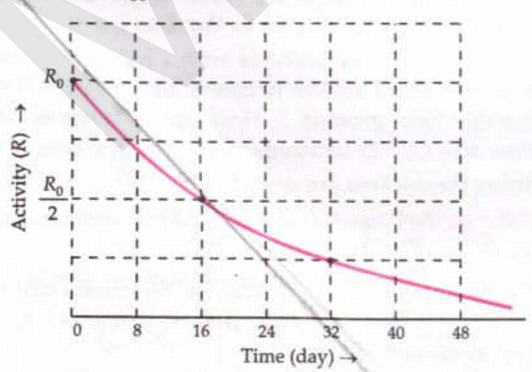

Fig. 13.18

The graph shows how the activity of a sample of radon-220 changes with time. Use the graph to determine its half-life. Calculate the value of decay constant of radon-220.

Solution. The decay process can be represented as follows :

$${}_{86}^{220}\text{Ru} \longrightarrow {}_{84}^{216}\text{Po} + {}_2^4\text{He}$$

The mass number and atomic number of Po and He have been indicated in the above equation.

The half-life of a radioactive sample is the time in which its activity is reduced to its half of its initial value. From the graph, the activity reduces from initial value R_0 to its half value $R_0 / 2$ in 16 days.

∴ Half-life of radon, $T_{1/2} = 16$ days

Decay constant, $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{16} = \mathbf{0.043\ day^{-1}}$.

Problem 25. **What are alpha particles ? In the reaction :**

$${}_Z^A X \longrightarrow \alpha + Y,$$

give the atomic number and mass number of Y.

[CBSE D 94C ; Haryana 02]

Solution. Alpha particles are helium nuclei, *i.e.*, doubly ionised helium atoms. They are represented as ${}_2^4$He

Given ${}_Z^A X \longrightarrow \alpha + Y$ or ${}_Z^A X \longrightarrow {}_2^4\text{He} + Y$

∴ Atomic number of $Y = Z - 2$

Mass number of $Y = A - 4$.

Problem 26. **The isotope of uranium ${}_{92}^{238}$U decays successively to form ${}_{90}^{234}$Th, ${}_{91}^{234}$Pa, ${}_{92}^{234}$U, ${}_{90}^{230}$Th, ${}_{88}^{226}$Ra, and ${}_{86}^{222}$Rn. What are the radiations emitted in each decay process ?** [CBSE D 95C]

Solution. The successive decays can be represented as

$${}_{92}^{284}\text{U} \xrightarrow{{}_2^4\text{He}} {}_{90}^{234}\text{Th} \xrightarrow{{}_{-1}^{0}e} {}_{91}^{234}\text{Pa} \xrightarrow{{}_{-1}^{0}e} {}_{92}^{234}\text{U}$$

$$\xrightarrow{{}_2^4\text{He}} {}_{90}^{234}\text{Th} \xrightarrow{{}_2^4\text{He}} {}_{88}^{226}\text{Ra} \xrightarrow{{}_2^4\text{He}} {}_{86}^{222}\text{Rn}$$

The radiations emitted in successive decays are : $\alpha, \beta^-, \beta^-, \alpha, \alpha, \alpha$.

Problem 27. **A neutron strikes a ${}_5^{10}$B nucleus with the subsequent emission of an alpha particle. Write the corresponding nuclear reaction. Find the atomic number, mass number and the chemical name of the remaining nucleus.** [CBSE OD 95C]

Solution. The nuclear reaction can be represented as

$${}_0^1n + {}_5^{10}\text{B} \longrightarrow {}_2^4\text{He} + {}_3^7\text{Li}$$

The remaining nucleus is ${}_3^7$Li. Its atomic number is 3 and mass number is 7.

Problem 28. **What is the difference between a photon and a neutrino ?**

Solution. A photon is one quantum of electromagnetic radiation having zero rest mass, no charge, zero spin and no antiparticle. It has a fixed energy which depends on frequency.

A neutrino is an elementary particle that accompanies a β-decay. It has zero rest mass, no charge and half spin. It has an antiparticle called antineutrino. It can have any energy from zero to any value permitted by the nuclear reaction. Its nature is non-electromagnetic.

Problem 29. **What is the role of control rods in a nuclear reactor ? Why are they made of cadmium ?** [NCERT]

Solution. For a controlled chain reaction, the average number of available neutrons should never exceed one per fission. Any excess neutrons over this critical unit should be absorbed. This is what the control rods do. They are made of cadmium because cadmium has a high cross-section for neutron absorption.

Problem 30. **What are delayed neutrons ? Explain the role of delayed neutrons in a nuclear reactor.** [CBSE D 91]

Solution. The neutrons produced by the subsequent decay of the initial fission fragments are called delayed neutrons. They are named so because they are produced after a fission – a few seconds later.

The production of delayed neutrons is crucial to mechanical control of the reactor. If all fission neutrons were produced instantly in fission, there would be no time for the minute adjustments required in a nuclear reactor to keep it critical.

Problem 31. **In a controlled thermal fusion reactor, what is the function of (*i*) the moderator, (*ii*) the control rods, (*iii*) the coolant, and (*iv*) the heavy water ?**

$$^{236}_{92}X \longrightarrow {}^{141}_{a}Y + {}^{b}_{36}Z + 3\,{}^{1}_{0}n$$

What are the values of the numbers a and b ? Calculate the total energy released per nuclear fission in MeV units when the masses in amu units are

of neutron = 1.009

of X-nucleus = 235.891

of Y-nucleus = 140.673

of Z-nucleus = 91.791 [CBSE OD 01C]

Solution. (*i*) A moderator slows down the fast moving neutrons produced during the fission.

(*ii*) Control rods absorb the excess neutrons and hence control the chain reaction.

(*iii*) The coolant removes the heat produced during fission and transfers it from the core of the reactor to the surroundings.

(*iv*) The heavy water acts both as a moderator and a coolant.

Given $^{236}_{92}X \longrightarrow {}^{141}_{a}Y + {}^{b}_{36}Z + 3\,{}^{1}_{0}n$

By conservation of mass,

$$141 + b + 3 \times 1 = 236$$
$$b = 236 - 144 = 92$$

By conservation of charge,

$$a + 36 + 3 \times 0 = 92$$
$$\therefore \quad a = 92 - 36 = 56$$

Mass defect

$$= m(X) - [m(Y) + m(Z) + 3m_n]$$
$$= 235.891 - [140.673 + 91.791 + 3 \times 1.009]$$
$$= 235.891 - 235.491 = 0.4 \text{ amu}$$
$$= 0.4 \times 931 \text{ MeV} = \mathbf{372.4 \text{ MeV}}.$$

Problem 32. **Neutrons produced in fission can be slowed down even by using ordinary water. Then, why is heavy water used for this purpose ?** [CBSE D 94]

Solution. Neutrons produced during fission get slowed if they collide with a nucleus of the same mass. As ordinary water contains hydrogen atoms (of mass nearly that of neutrons), so it can be used as a moderator. But it absorbs neutrons at a fast rate via the reaction :

$$n + p \longrightarrow d + \gamma$$

Here d is deuteron. To overcome this difficulty, heavy water is used as a moderator which has negligible cross-section for neutron absorption.

Problem 33. **A chain reaction dies out sometimes. Why ?**

Solution. A chain reaction may die out due to any of the following causes :

1. Excessive neutron leakage if the size of the fissionable material is smaller than the critical size.
2. Fast neutrons may escape the fissionable material without causing further fissions.
3. Some neutrons may suffer non-fission capture by $^{238}_{92}$U nuclei.

Problem 34. **Give one similarity and one dissimilarity between nuclear fission and nuclear fusion.** [Punjab 10, 11]

Solution. ***Similarity***. Both nuclear fission and fusion are the source of enormous amount of energy.

Dissimilarity. In nuclear fission, a heavy nucleus splits into two smaller nuclei. In nuclear fusion, two smaller nuclei fuse together to form a heavier nucleus.

Problem 35. **Give two points of difference between nuclear fission and nuclear fusion.** [CBSE OD 96C]

Solution.

Nuclear fission	Nuclear fusion
1. A heavy nucleus when excited splits up into two smaller nuclei of comparable masses.	Two nuclei fuse together to form a heavier nucleus.
2. Conditions of high temperature and pressure are not necessary for its occurrence.	Conditions of extremely high pressure and temperature are necessary for its occurrence.

Problem 36. Safety of nuclear reactors is an important issue that has attracted much attention recently. Guess some of the safety problems that a nuclear engineer must cope with in reactor design.

Solution. A major safety problem arises from the fact that the nuclear waste from the reactor contains some long-lived radioactive isotopes. Besides, appropriate cooling systems have to be designed to prevent accidents due to excessive heating (and melting) of the reactor core.

Problem 37. What is nuclear fall out ? How can it be reduced ?

Solution. The nuclear explosion is accompanied by the emission of penetrating radiations like X-rays, ultraviolet rays, α-particles, β-particles, γ-rays and a large number of radioisotopes. All these radiations and particles constitute nuclear fall out which is very harmful to the living beings. The fall out can be reduced by carrying nuclear explosion deep inside the earth, that also in deserts.

Problem 38. Name the reaction which takes place when a slow neutron beam strikes $^{235}_{92}$U nuclei. Write the nuclear reaction involved. [CBSE D 03]

Solution. When a slow neutron strikes $^{235}_{92}$U nucleus, nuclear fission takes place.

$$^{235}_{92}\text{U} + ^{1}_{0}n \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3\,^{1}_{0}n + \text{Energy}$$

Problem 39. Calculate the binding energy per nucleon (in MeV) for $^4_2$He and $^3_2$He. Comment on the difference of these binding energies and its significance in relation to α-decay of the nuclei.

[Given : mass of $^1_1$H = 1.00783 u,
mass of $^1_0$n = 1.00867 u,
mass of $^3_2$He = 3.01664 u,
mass of $^4_2$He = 4.00387 u] [CBSE D 05C]

Solution. B.E. of $^4_2$He $= [2m_p + 2m_n - m(^4_2\text{He})] \times c^2$

$= [2 \times 1.00783 + 2 \times 1.00867 - 4.00387] \times 931$ MeV

$= [4.03390 - 4.00387] \times 931 = 0.02933 \times 931$ MeV

$= 27.30623$ MeV

B.E. per nucleon of $^4_2$He

$= \frac{27.30623}{4} = \mathbf{6.83\ MeV}$

B.E. of $^3_2$He

$= [2m_p + m_n - m(^3_2\text{He})]\,c^2$

$= [2 \times 1.00783 + 1.00867 - 3.01664] \times 931$ MeV

$= 0.00769 \times 931$ MeV $= 7.16$ MeV

B.E. per nucleon of $^3_2$He

$= \frac{7.16}{3} = \mathbf{2.39\ MeV}$

As the binding energy per nucleon of $^4_2$He is larger than that of $^3_2$He, so unstable heavy nuclei prefer to get stabilised through α-decay.

Problem 40. A nucleus makes a transition from one permitted energy level to another level of lower energy. Name the region of the electromagnetic spectrum to which the emitted photon belongs. What is the order of its energy in electron volts ? Write four characteristics of nuclear force. [CBSE Sample Paper 05]

Solution. In a nuclear transition, short wavelength electromagnetic waves called gamma rays are emitted. The emitted photons have energy of the order of million electron volts.

Characteristics of nuclear forces are :

(*i*) short range,
(*ii*) very strong,
(*iii*) charge independent, and
(*iv*) non-central forces.

Problem 41. Define mass number (A) of an atomic nucleus. Assuming the nucleus to be spherical, give the relation between mass number (A) and the radius (R) of the nucleus.

Calculate the density of nuclear matter. Radius of nucleus of $^1_1$H = 1.1×10^{-15} Å. What is the ratio of the order of magnitude of density of nuclear matter and density of ordinary matter ? [CBSE Sample Paper 05]

Solution. The total number of protons and neutrons present inside a nucleus is called its mass number (A).

The relation between the mass number (A) and radius (R) of the nucleus is

$$R = R_0\,A^{1/3}, \quad \text{where } R_0 = 1.1 \times 10^{-15}\text{ m}.$$

Density of nuclear matter,

$$\rho = \frac{\text{Mass of } ^1_1\text{H}}{\text{Volume}} = \frac{1.66 \times 10^{-27}\text{ kg}}{\frac{4}{3}\pi(1.1 \times 10^{-15}\text{m})^3}$$

$$= 3 \times 10^{17}\text{ kgm}^{-3}$$

$$\frac{\text{Density of nuclear matter}}{\text{Density of ordinary matter}} = \frac{10^{17}}{10^3} = \mathbf{10^{14}}\,.$$

Problem 42. Define the term 'Activity' of a radioactive substance. State its SI unit. Two different radioactive elements with half lives T_1 and T_2 have N_1 and N_2 (undecayed) atoms respectively present at a given instant. Determine the ratio of their activities at this instant. [CBSE Sample Paper 08]

Solution. The activity of a sample is defined as the number of radioactive disintegrations taking place per second at any instant in the sample. Its SI unit is becquerel.

1 becquerel = 1 Bq = 1 decay per second.

$$R = \frac{dN}{dt} = \lambda N = \frac{0.693}{T_{1/2}} N$$

For sample 1, $\quad R_1 = \frac{0.693}{T_1} N_1$

For sample 2, $\quad R_2 = \frac{0.693}{T_2} N_2$

$$\therefore \quad \frac{R_1}{R_2} = \frac{N_1 T_2}{N_2 T_1}$$

Problem 43. Prove that the instantaneous rate of change of the activity of a radioactive substance is inversely proportional to the square of its half life. [CBSE Sample Paper 08]

Solution. Instantaneous activity, $\quad R = -\frac{dN}{dt} = \lambda N$

The instantaneous rate of change of the activity of a radioactive substance,

$$\frac{dR}{dt} = \frac{d}{dt}(\lambda N) = \lambda \frac{dN}{dt} = \lambda(-\lambda N) = -\lambda^2 N = -\left(\frac{\log_e 2}{T_{1/2}}\right)^2 N$$

$$\therefore \quad \frac{dR}{dt} \propto \frac{1}{(T_{1/2})^2}$$

Problem 44. Define the activity of a radionuclide. Write its SI unit. Give a plot of the activity of a radioactive species versus time.

How long will a radioactive iostope, whose half life is T years, take for its activity to reduce to 1/8th of its initial value ? [CBSE OD 09, D10]

Solution. Refer to the solution of Problem 42 above.

The activity R decreases exponentially with time t ($R = R_0 e^{-\lambda t}$) as shown in the graph.

$$\frac{R}{R_0} = \left(\frac{1}{2}\right)^n$$

or $\quad \frac{1}{8} = \left(\frac{1}{2}\right)^{t/T}$

or $\quad \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{t/T}$

$\therefore \quad t = 3T$.

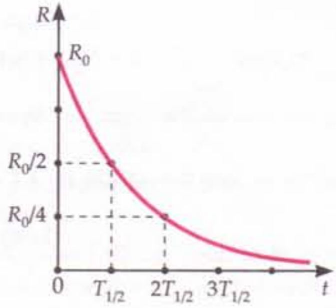

Fig. 13.19

Problem 45. The potential energy (V), of a pair of nucleons varies with separation (r) between them, in the manner shown in Fig. 13.20.

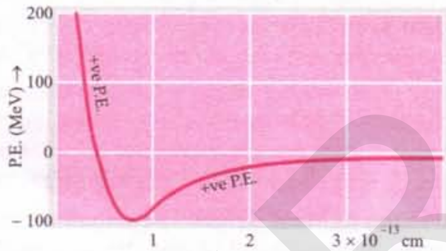

Fig. 13.20

Use this graph to explain why the force between the nucleons must be regarded as

(i) strongly repulsive for separation values less than r_0.

(ii) attractive nuclear force ($r > r_0$).

Solution. The potential energy of the two nucleons is minimum at a separation of r_0.

(i) For separation values less than r_0, the P.E. increases rapidly with the decrease in separation r. This indicates strong repulsion between the nucleons.

(ii) For separation values greater than r_0, the P.E. is negative which falls to zero for a separation more than a few femtometers. This indicates an attractive force between the nucleons.

Problem 46. Why is it necessary to slow down the neutrons, produced through the fission of $^{235}_{92}U$ nuclei (by neutrons), to sustain a chain reaction ? What type of nuclei are (preferably) needed for slowing down neutrons ? [CBSE OD 08C]

Solution. Fast neutrons may escape the fissionable material without causing further fission. Hence they are slowed down to thermal velocities by using moderators like heavy water, graphite, beryllium, etc., which are rich in hydrogen nuclei or protons.

Problem 47. Give reasons for (a) Lighter elements are better moderators for a nuclear reactor than heavier elements. (b) Very high temperatures as those obtained in the interior of the sun are required for fusion reaction to take place.

Solution. (a) The nucleus of a lighter element contains a relatively larger number of protons. When fast neutrons are passed through such an element, they make elastic collisions with its protons, which have smaller velocities. After few interactions, the final velocities of the neutrons become equal to the low velocities of protons and hence they get slowed down.

(b) To overcome coulomb repulsion, the fusing nuclei are given enough thermal energy by raising their temperature to $10^6 - 10^7$ K. Such a high temperature is available in the interior of the sun.

GUIDELINES TO NCERT EXERCISES

You may find the following data useful in solving the exercises :

e	=	1.6×10^{-19}C
$1/(4\pi\varepsilon_0)$	=	9×10^{9} Nm^2/C
1 MeV	=	1.6×10^{-13} J
1 year	=	3.154×10^{7} s
m_H	=	1.007825 amu

$m(^4_2\text{He})$	=	4.002603 amu
N	=	6.023×10^{23} per mole
k	=	1.381×10^{-23} JK^{-1}
1 amu	=	931.5 MeV
m_n	=	1.008665 amu

13.1. (*a*) *Two stable isotopes of lithium $^6_3$Li and $^7_3$Li have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 amu and 7.01600 amu respectively. Find the atomic weight of lithium.*

(*b*) *Boron has two stable isotopes, $^{10}_5$B and $^{11}_5$B. Their respective masses are 10.01294 amu and 11.00931 amu, and the atomic weight of boron is 10.811 amu. Find the abundances of $^{10}_5$B and $^{11}_5$B.*

Ans. The atomic weight of lithium is

$$m(\text{Li}) = \frac{7.5\times6.01512 + 92.5\times7.01600}{100}$$

$$= \frac{45.1134 + 648.98}{100} = \frac{694.0934}{100}$$

$$\approx \mathbf{6.941\ amu.}$$

(*b*) Suppose the natural boron contains x% of $^{10}_5$B isotope and $(100-x)$% of $^{11}_5$B isotope. Then,

Atomic mass of natural boron

= Weighted average of the masses of two isotopes

$$\therefore \quad 10.811 = \frac{x\times10.01294 + (100-x)\times11.00931}{100}$$

or $\quad 1081.1 = -0.99637x + 1100.931$

or $\quad x = \dfrac{19.831}{0.99637} = 19.9$

$\therefore$ Relative abundance of $^{10}_5$B isotope = **19.9%.**

Relative abundance of $^{11}_5$B isotope = **80.1%.**

13.2. *The three stable isotopes of neon : Ne^{20}, Ne^{21}, Ne^{22} have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 amu, 20.99 amu and 21.99 amu respectively. Obtain the average atomic mass of neon.*

Ans. The average atomic mass of neon is

$$m(\text{Ne}) = \frac{90.51\times19.99 + 0.27\times20.99 + 9.22\times21.99}{100}$$

$$= \frac{1736.89 + 5.67 + 202.75}{100}$$

$$= \frac{1945.31}{100} = \mathbf{20.18\ amu.}$$

13.3. *Obtain the binding energy of a nitrogen nucleus ($^{14}_7$N) from the following data :*

$m_H = 1.00783$ amu ; $m_n = 1.00867$ amu ; $m_N = 14.00307$ amu.

Give your answer in MeV.

Ans. The $^{14}_7$N nucleus contains 7 protons and 7 neutrons.

Mass of 7 protons $= 7\times1.00783 =$ 7.05481 amu

Mass of 7 neutrons $= 7\times1.00867 =$ 7.06069 amu

Total mass = 14.11550 amu

Mass of $^{14}_7$N nucleus = 14.00307 amu

Mass defect, $\Delta m =$ 0.11243 amu

B.E. of nitrogen nucleus

$= 0.11243\times931.5 =$ **104.7 MeV.**

13.4. *Obtain the binding energy of the nuclei $^{56}_{26}$Fe and $^{209}_{83}$Bi in units of MeV from the following data :*

$m_H = 1.007825$ amu

$m_n = 1.008665$ amu

$m(^{56}_{26}Fe) = 55.934939$ amu

$m(^{209}_{83}Bi) = 208.980388$ amu

1 amu = 931.5 MeV

Which nucleus has greater binding energy per nucleon ?

Ans. The $^{56}_{26}$Fe nucleus contains 26 protons and 30 neutrons.

Mass of 26 protons

$= 26\times1.007825 = 26.203450$ amu

Mass of 30 neutrons

$= 30\times1.008665 = 30.259950$ amu

Total mass = 56.463400 amu

Mass of $^{56}_{26}$Fe nucleus = 55.934939 amu

Mass defect, $\Delta m = 0.528461$ amu

B.E. of $^{56}_{26}$Fe nucleus

$= \Delta m\times931.5$ MeV $= 0.528461\times931.5$

= **492.26 MeV**

B.E./nucleon of $^{56}_{26}Fe = \frac{492.26}{56} = \mathbf{8.79\ MeV.}$

Now, the $^{209}_{83}Bi$ nucleus contains 83 protons and 126 neutrons.

Mass of 83 protons

$$= 83 \times 1.007825 = 83.649475 \text{ amu}$$

Mass of 126 neutrons

$$= 126 \times 1.008665 = 127.091790 \text{ amu}$$

Total mass $= 210.741265$ amu

Mass of $^{209}_{83}Bi$ nucleus $= 208.980388$ amu

Mass defect, $\Delta m = 1.760877$ amu

B.E. of $^{209}_{83}Bi$ nucleus $= 1.760877 \times 931.5$

$$= \mathbf{1640.3\ MeV}$$

B.E./nucleon of $^{209}_{83}Bi = \frac{1640.3}{209} = \mathbf{7.85\ MeV}$

Clearly, $^{56}_{26}Fe$ has a greater B.E. per nucleon. In fact, it is the maximum value.

13.5. *A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}Cu$ atoms (of mass 62.92960 amu). The masses of proton and neutron are 1.00783 amu and 1.00867 amu, respectively.*

Ans. The $^{63}_{29}Cu$ nucleus contains 29 protons and 34 neutrons.

Mass of 29 protons $= 29 \times 1.00783 = 29.22707$ amu

Mass of 34 protons $= 34 \times 1.00867 = 34.29478$ amu

Total mass $= 63.52185$ amu

Mass of $^{63}_{29}Cu$ nucleus $= 62.92960$ amu

Mass defect, $\Delta m = 0.59225$ amu

B.E. of $^{63}_{29}Cu$ nucleus

$$= 0.59225 \times 931.5 \text{ MeV} = 551.5032 \text{ MeV}$$

Number of atoms in 63 g of Cu

$$= \text{Avogadro's number} = 6.023 \times 10^{23}$$

$\therefore$ Number of atoms in 3 g of Cu

$$= \frac{6.023 \times 10^{23} \times 3}{63} = 2.868 \times 10^{22}$$

Energy required to separate all the neutrons and protons from each other of 3 g copper coin

$$= 551.5032 \times 2.868 \times 10^{22} = \mathbf{1.582 \times 10^{25}\ MeV.}$$

13.6. *Write nuclear reaction equations for*

(*i*) *α-decay of* $^{226}_{88}Ra$ (*ii*) *α-decay of* $^{242}_{94}Pu$

(*iii*) *β^--decay of* $^{32}_{15}P$ (*iv*) *β^--decay of* $^{210}_{83}Bi$

(*v*) *β^+-decay of* $^{11}_{8}C$ (*vi*) *β^+-decay of* $^{97}_{43}Tc$

(*vii*) *Electron capture of* $^{120}_{54}Xe$

Ans.

(*i*) $^{226}_{88}Ra \longrightarrow ^{222}_{86}Rn + ^{4}_{2}He$

(*ii*) $^{242}_{94}Pu \longrightarrow ^{238}_{92}U + ^{4}_{2}He$

(*iii*) $^{32}_{15}P \longrightarrow ^{32}_{16}S + e^- + \bar{\nu}$

(*iv*) $^{210}_{83}B \longrightarrow ^{210}_{84}Po + e^- + \bar{\nu}$

(*v*) $^{11}_{6}C \longrightarrow ^{11}_{5}B + e^+ + \nu$

(*vi*) $^{97}_{43}Tc \longrightarrow ^{97}_{42}Mo + e^+ + \nu$

(*vii*) $^{120}_{54}Xe + e^+ \longrightarrow ^{120}_{53}I + \nu$

13.7. *A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to (a) 3.125%, (b) 1% of its original value ?*

Ans. (*a*) $\frac{R}{R_0} = \frac{N}{N_0} = \frac{3.125}{100} = \frac{1}{32}$

or $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^5$ or $n = 5$

$\therefore$ $t = nT = \mathbf{5\,T\ years.}$

(*b*) $\frac{R}{R_0} = \frac{N}{N_0} = \frac{1}{100}$

Required time,

$$t = \frac{2.303}{\lambda} \log \frac{N_0}{N} = \frac{2.303\,T}{0.693} \log 100$$

$$= \frac{2.303 \times 2 \times T}{0.693} \approx \mathbf{6.65\,T\ years.}$$

13.8. *The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive C^{14} present with the ordinary carbon isotope C^{12}. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half life (=5730 years) of C^{14}, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of C^{14} dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.*

Ans. Given normal activity,

$$R_0 = 15 \text{ decays min}^{-1}$$

Present activity,

$$R = 9 \text{ decays min}^{-1},$$

$$T_{1/2} = 5730 \text{ years}$$

Since activity is proportional to the number of radioactive atoms, therefore,

$$\frac{R}{R_0} = \frac{N}{N_0} = \frac{N_0\, e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

or $\frac{9}{15} = e^{-\lambda t}$ or $e^{\lambda t} = \frac{15}{9}$

Taking natural logarithms,

$$\log_e e^{\lambda t} = \log_e \frac{15}{9}$$

or $$\lambda t \log_e e = 2.303 \log_{10} \frac{5}{3} = 2.303 \times 0.2218$$

or $$t = \frac{0.5109}{\lambda} \quad [\because \log_e e = 1]$$

As $$T_{1/2} = \frac{0.693}{\lambda}$$

$$\therefore \quad t = \frac{0.5109}{0.693/T_{1/2}} = \frac{0.5109}{0.693} \times T_{1/2}$$

$$= \frac{0.5109 \times 5730}{0.693} \text{ years} = \mathbf{4224 \text{ years.}}$$

13.9. *Obtain the amount of $^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}Co$ is 5.3 years.*

Ans. Here $R = 8.0$ mCi

$$= 8.0 \times 10^{-3} \times 3.7 \times 10^{10} \text{ dis s}^{-1}$$
$$= 29.6 \times 10^7 \text{dis s}^{-1}$$
$$T_{1/2} = 5.3 \text{ years} = 5.3 \times 3.16 \times 10^7 \text{ s}$$

But $$R = \lambda N = \frac{0.693}{T_{1/2}} . N$$

$$\therefore \quad N = \frac{RT_{1/2}}{0.693} = \frac{29.6 \times 10^7 \times 5.3 \times 3.16 \times 10^7}{0.693}$$
$$= 7.15 \times 10^{16} \text{ atoms}$$

As 60 g of cobalt contains 6.023×10^{23} atoms, so the amount necessary to obtain a source of the required strength

$$= \frac{60 \times 7.15 \times 10^{16}}{6.023 \times 10^{23}} = \mathbf{7.123 \times 10^{-6} \text{ g.}}$$

13.10. *The half-life of $^{90}_{38}Sr$ is 28 years. What is the disintegration rate of 15 mg of this isotope ?*

Ans. Here $T_{1/2} = 28 \text{ years} = 28 \times 3.154 \times 10^7$ s

$$m = 15 \text{ mg} = 0.015 \text{ g}, \; M = 90$$

Number of atoms in 0.015 g sample of $^{80}_{38}$Sr,

$$N = \frac{m}{M} \times \text{Avogadro's number}$$
$$= \frac{0.015 \times 6.023 \times 10^{23} \text{ atoms}}{90}$$

Activity of the sample,

$$R = \lambda N = \frac{0.693}{T_{1/2}} . N = \frac{0.693 \times 0.015 \times 6.023 \times 10^{23}}{28 \times 3.154 \times 10^7 \times 90}$$
$$= 7.877 \times 10^{10} \text{ disintegrations/second}$$
$$= \mathbf{7.877 \times 10^{10}} \text{ Bq}$$
$$= \frac{7.877 \times 10^{10}}{3.7 \times 10^{10}} \text{ Ci} = \mathbf{2.13 \text{ Ci.}}$$

13.11. *Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}Au$ and the silver isotope $^{107}_{47}Au$. What is the approximate ratio of their nuclear mass densities ?*

Ans. As $R = R_0 A^{1/3}$, where $R_0 = 1.1 \times 10^{-15}$ m

$$\therefore \quad \frac{R(^{197}\text{Au})}{R(^{107}\text{Ag})} = \left(\frac{197}{107}\right)^{1/3} \simeq \mathbf{1.23}$$

Since the nuclear mass density is independent of the size of the nucleus, so

$$\frac{\rho_{nu}(\text{Au})}{\rho_{nu}(\text{Ag})} \simeq \mathbf{1}$$

13.12. *Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of*

(a) $^{226}_{88}Ra$ (b) $^{220}_{86}Rn$.

Given
$$m(^{226}_{88}Ra) = 226.02540 \text{ amu},$$
$$m(^{222}_{86}Rn) = 222.01750 \text{ amu},$$
$$m(^{220}_{86}Rn) = 220.01137 \text{ amu},$$
$$m(^{216}_{84}Po) = 216.00189 \text{ amu}.$$

Ans. (a) $^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He} + \text{Q}$

$$Q = [m(^{226}_{88}\text{Ra}) - m(^{222}_{86}\text{Rn}) - m(^{4}_{2}\text{He})] c^2$$
$$= [226.02540 - 222.01750 - 4.00260] \times 931.5 \text{ MeV}$$
$$= 0.0053 \times 931.5 = \mathbf{4.937 \text{ MeV}}$$
$$K_\alpha = \frac{A-4}{A} Q$$
$$= \frac{226-4}{226} \times 4.937 = \mathbf{4.85 \text{ MeV.}}$$

(b) $^{220}_{86}\text{Rn} \longrightarrow {}^{216}_{84}\text{Po} + {}^{4}_{2}\text{He} + \text{Q}$

$$Q = [m(^{220}_{86}\text{Rn}) - m(^{216}_{84}\text{Po}) - m(^{4}_{2}\text{He})] c^2$$
$$= [220.01137 - 216.00189 - 4.00260] \times 931.5 \text{ MeV}$$
$$= 0.00688 \times 931.5$$
$$= \mathbf{6.41 \text{ MeV}}$$
$$K_\alpha = \frac{A-4}{A} Q$$
$$= \frac{220-4}{220} \times 6.41 = \mathbf{6.29 \text{ MeV.}}$$

13.13. *The radionuclide ^{11}C decays according to*

$$^{11}_{6}C \longrightarrow {}^{11}_{5}B + e^+ + \nu : T_{1/2} = 20.3 \text{ min}.$$

The maximum energy of the emitted positron is 0.960 MeV. Given the mass values :

$$m(^{11}_{6}\text{C}) = 11.011434 \text{ amu}$$
$$m(^{11}_{5}\text{B}) = 11.009305 \text{ amu}$$
$$m_e = 0.000548 \text{ amu}$$

Calculate Q and compare it with the maximum energy of the positron emitted.

Ans. $^{11}_{6}C \longrightarrow ^{11}_{5}B + e^{+} + \nu + Q$

where Q is the energy released in the decay process. It is given by

$$Q = [m_N(^{11}_{6}C) - m_N(^{4}_{5}B) - m_e]\,c^2$$

To express Q-value in terms of atomic masses, we have to subtract $6m_e$ from the atomic mass of carbon and $5m_e$ from the atomic mass of boron to get the corresponding nuclear masses. So we get

$$Q = [m(^{11}C) - 6m_e - m(^{11}B) + 5m_e - m_e]\,c^2$$
$$= [m(^{11}_{6}C) - m(^{11}_{5}B) - 2m_e]\,c^2$$
$$= [11.011434 - 11.009305 - 2 \times 000548]\text{ amu} \times c^2$$
$$= 0.001033\text{ amu} \times 931.5\,\frac{\text{MeV}}{\text{amu}}$$
$$= 0.9622\text{ MeV} \propto \mathbf{0.96\ MeV}.$$
$$Q = E_d + E_e + E_\nu$$

The daughter nucleus is much heavier than e^+ and ν, so its energy $E_d = 0$. When the energy of neutrino is minimum, the energy of position is maximum and $E_e \simeq Q$.

13.14. *The nucleus Ne^{23} decays by β-emission. Write down the β-decay equation and determine the maximum kinetic energy of the electrons emitted from the following data :*
$m(^{23}_{10}Ne) = 22.994466$ amu, $m(^{23}_{11}Na) = 22.989770$ amu.

[CBSE D 08]

Ans. The β-decay of $^{23}_{10}Ne$ may be represented as

$$^{23}_{10}Ne \longrightarrow ^{23}_{11}Na + ^{0}_{-1}e + \bar{\nu} + Q$$

Ignoring the rest mass of neutrino, the expression for the kinetic energy released may be written as

$$Q = [m_N(^{23}_{10}Ne) - m_N(^{23}_{11}Na) - m_e]c^2$$
$$= [\{m_N(^{23}_{10}Ne) + 10m_e\} - \{m_N(^{23}_{11}Na) + 11m_e\}]c^2$$
$$= [m(^{23}_{10}Ne) - m(^{23}_{11}Na)]c^2$$
$$[\because c^2 = 931.5\text{ MeV/amu}]$$
$$= [22.994466 - 22.989770] \times 931.5\text{ MeV}$$
$$= 0.004696 \times 931.5\text{ MeV} = \mathbf{4.374\ MeV.}$$

As ^{23}Na is massive, the kinetic energy released is mainly shared by electron-positron pair. When the neutrino carries no energy, the electron has a maximum kinetic energy equal to 4.374 MeV.

13.15. *The Q value of a nuclear reaction $A + b \rightarrow C + d$ is defined by* $Q = [m_A + m_b - m_C - m_d]\,c^2$ *where the masses refer to nuclear rest masses. Determine from the given data whether the following reactions are exothermic or endothermic.* **[CBSE OD 14C]**

(i) $^{1}_{1}H + ^{3}_{1}H \rightarrow ^{2}_{1}H + ^{2}_{1}H$

(ii) $^{12}_{6}C + ^{12}_{6}C \rightarrow ^{20}_{10}Ne + ^{4}_{2}He$

Atomic masses are given to be :

$m(^{1}_{1}H) = 1.007825$ amu

$m(^{2}_{1}H) = 2.014102$ amu

$m(^{3}_{1}H) = 3.016049$ amu

$m(^{12}_{6}H) = 12.000000$ amu

$m(^{20}_{10}H) = 19.992439$ amu

$m(^{4}_{2}H) = 4.002603$ amu

Ans. (i) $^{1}_{1}H + ^{3}_{1}H \longrightarrow ^{2}_{1}H + ^{2}_{1}H$

$$Q = [m(^{1}_{1}H) + m(^{3}_{1}H) - \{m(^{2}_{1}H) + m(^{2}_{1}H)\}]\,c^2$$
$$= [(1.007825 + 3.016049) - 2 \times 2.014102] \times 931.5\text{ MeV}$$
$$= (4.023874 - 4.028204) \times 931.5\text{ MeV}$$
$$= -0.00433 \times 931.5\text{ MeV} = \mathbf{-4.033\ MeV}$$

Negative value of Q indicates that the reaction is *endothermic.*

(ii) $^{12}_{6}C + ^{12}_{6}C \longrightarrow ^{20}_{10}Ne + ^{4}_{2}He$

$$Q = [2m(^{12}_{6}C) - \{m(^{20}_{10}Ne) + m(^{4}_{2}He)\}]\,c^2$$
$$= [2 \times 12.000000 - (19.992439 + 4.002603)] \times 931.5\text{ MeV}$$
$$= (24 - 23.995042) \times 931.5\text{ MeV}$$
$$= 0.004958 \times 931.5\text{ MeV} = \mathbf{4.618\ MeV}$$

Positive value of Q indicates that the reaction is *exothermic.*

13.16. *Suppose, we think of fission of a $^{56}_{26}Fe$ nucleus into two equal fragments, $^{28}_{13}Al$. Is the fission energetically possible ? Argue by working out Q of the process. Given $m(^{56}_{26}Fe) = 55.93494$ amu and $m(^{28}_{13}Al) = 27.98191$ amu.*

Ans. $^{56}_{26}Fe \longrightarrow ^{28}_{13}Al + ^{28}_{13}Al + Q$

$$Q = [m(^{56}_{26}Fe) - 2m(^{28}_{13}Al)]\,c^2$$
$$= [55.93494 - 2 \times 27.98191] \times 931.5\text{ MeV}$$
$$= -0.02888 \times 931.5 = -26.90\text{ MeV}$$

As the Q-value is negative, the fission is not possible energetically.

13.17. *The fission properties of $^{239}_{94}Pu$ are very similar to those of $^{235}_{92}U$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure $^{239}_{94}Pu$ undergo fission ?*

Ans. Number of atoms present in 239 g of $^{239}_{94}Pu$

$$= 6.023 \times 10^{23}$$

∴ Number of atoms present in 1 kg or 1000 g of $^{239}_{94}Pu$

$$= \frac{6.023 \times 10^{23} \times 1000}{239} = 2.52 \times 10^{24}$$

Energy released per fission = 180 MeV

Total energy released

$$= 2.52 \times 10^{24} \times 180\text{ MeV} = \mathbf{4.54 \times 10^{26}\ MeV}.$$

13.18. *A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much $^{235}_{92}U$ did it contain initially ? Assume the reactor operates 80% of the time, that all the energy generated arises from the fission of $^{235}_{92}U$ and that this nuclide is consumed only by the fission process.*

Ans. Power of the reactor, $P = 1000\text{ MW} = 10^9\text{ W}$

Time of power generation,

$$t = 5y = 5 \times 3.154 \times 10^7 \text{s}$$

Total energy generated in 5y with 80% on-time

$$= 80\% \text{ of } Pt = 0.8 \times 10^9 \times 5 \times 3.154 \times 10^7 \text{J}$$

Energy generated in each fission of $^{235}_{92}U$

$$= 200\text{ MeV} = 200 \times 1.6 \times 10^{-13}\text{J}$$

Number of atoms in 235 g of $^{235}_{92}U = 6 \times 10^{23}$

Number of atoms in 1 g of $^{235}_{92}U = \dfrac{6 \times 10^{23}}{235}$

Energy generated per gram of $^{235}_{92}U$

$$= \frac{200 \times 1.6 \times 10^{-13} \times 6 \times 10^{23}}{235}\text{ J g}^{-1}$$

The amount of $^{235}_{92}U$ consumed in 5y with 80% on-time

$$= \frac{\text{Total energy generated}}{\text{Energy generated per gram}}$$

$$= \frac{0.8 \times 10^9 \times 5 \times 3.154 \times 10^7 \times 235}{200 \times 1.6 \times 10^{-13} \times 6 \times 10^{23}}\text{ g}$$

$$= 1.544 \times 10^6 \text{ g} = 1544 \text{ kg.}$$

This amount is half the fuel taken initially.

$\therefore$ Mass of $^{235}_{92}U$ taken initially = **3088 kg.**

13.19. *How long an electric lamp of 100 W can be kept glowing by fusion of 2.0 kg of deuterium ? The fusion reaction can be taken as*

$$^2_1H + ^2_1H \longrightarrow ^3_2He + n + 3.2\text{ MeV}.$$

Ans. Number of atoms present in 2 g of deuterium

$$= 6 \times 10^{23}$$

Number of atoms present in 2.0 kg or 2000 g of deuterium

$$= \frac{6 \times 10^{23} \times 2000}{2} = 6 \times 10^{26}$$

Energy released in the fusion of 2 deuterium atoms

$$= 3.2\text{ MeV}$$

Total energy released in the fusion of 2.0 kg of deuterium atoms

$$= \frac{3.2}{2} \times 6 \times 10^{26} = 9.6 \times 10^{26}\text{ MeV}$$

$$= 9.6 \times 10^{26} \times 1.6 \times 10^{-13}\text{J}$$

$$= 15.34 \times 10^{13}\text{ J}$$

Energy consumed by the bulb per second

$$= 100\text{ J}$$

Time for which the bulb will glow

$$= \frac{15.34 \times 10^{13}}{100}\text{ s} = \frac{15.34 \times 10^{11}}{3.15 \times 10^7}\text{ years}$$

$$= \mathbf{4.9 \times 10^4 \text{ years.}}$$

13.20. *Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint. The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)*

Ans. Charge on each deuteron,

$$e = 1.6 \times 10^{-19}\text{C}$$

Radius of deuteron,

$$R = 2.0\text{ fm} = 2.0 \times 10^{-15}\text{ m}$$

The Coulomb barrier is given by

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{2R}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{2 \times 2 \times 10^{-15}}\text{ J}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15} \times 1.6 \times 10^{-16}}\text{ keV} = \mathbf{360\text{ keV}}.$$

13.21. *From the relation $R = R_0 A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e., independent of A).*

Ans. Refer answer to Q. 6 on page 13.4.

13.22. *For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture i.e., electron from an inner orbit (say the K - Shell) is captured by the nucleus and a neutrino is emitted :*

$$e^- + ^A_ZX \longrightarrow {}_{Z-1}^{A}Y + \nu$$

Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice versa.

Ans. Consider the two competing processes :

Positron emission :

$$^A_ZX \longrightarrow {}_{Z-1}^{A}Y + e^+ + \nu + Q_1$$

Electron capture :

$$e^- + ^A_ZX \longrightarrow {}_{Z-1}^{A}Y + \nu + Q_2$$

The energy changes in the two processes are :

$$Q_1 = [m_N(^A_ZX) - m_N({}_{Z-1}^{A}Y) - m_e]c^2$$

$$= [m(^A_ZX) - Zm_e - m({}_{Z-1}^{A}Y) + (Z-1)m_e - m_e]c^2$$

$$= [m(^A_ZX) - m({}_{Z-1}^{A}Y) - 2m_e]c^2$$

$$Q_2 = [m_N(^A_ZX) + m_e - m_N({}_{Z-1}^{A}Y)]c^2$$

$$= [m(^A_ZX) - m({}_{Z-1}^{A}Y)]c^2$$

This means $Q_1 > 0$ implies $Q_2 > 0$ but $Q_2 > 0$ does not necessarily imply $Q_1 > 0$. Thus if positron emission is energetically allowed, electron capture is necessarily allowed but not *vice versa.*

13.23. *In a periodic table, the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative* natural abundance on Earth. The three isotopes and their *masses are* $^{24}_{12}Mg$ *(23.98504 u),* $^{25}_{12}Mg$ *(24.98564 u),* $^{26}_{12}Mg$ *(25.98259 u). The natural abundance of* $^{24}_{12}Mg$ *is 78.99% by mass. Calculate the abundances of the other two isotopes.*

Ans. The abundance of $^{24}_{12}$Mg (23.98504 u) is 78.99%.

Then abundance of $^{25}_{12}$Mg (24.98564 u) is [100 – (x + 78.99 u)]%, where x% is the abundance of $^{26}_{12}$Mg (25.98259).

Average atomic mass of Mg

= Weighted average of masses of isotopes

$$24.312 = \frac{78.99 \times 23.98504 + [100-(x+78.99)] \times 24.98564 + x\,[25.98259]}{100}$$

$$2431.2 = 1894.5783 + 2498.564 + x \times 0.99615 - 1973.6157$$

or $$x = \frac{4404.8157 - 4393.1423}{0.99695}$$

$$= \frac{11.6734}{0.99695} = 11.71$$

$\therefore \quad x = 11.71\%$

Abundance of $^{26}_{12}$Mg = **11.71%**

Abundance of $^{25}_{12}$Mg = 100 – (11.71 + 78.99) = **9.3%**.

13.24. *The neutron separation energy is defined to be the energy required to remove a neutron from a nucleus. Obtain the neutron separation energies of the nuclei* $^{41}_{20}Ca$ *and* $^{27}_{13}Al$ *from the following data :*

$$m_n = 1.008665 \text{ amu};$$

$$m(^{40}_{20}Ca) = 39.962591 \text{ amu};$$

$$m(^{41}_{20}Ca) = 40.962278 \text{ amu};$$

$$m(^{26}_{13}Al) = 25.986895 \text{ amu};$$

$$m(^{27}_{13}Al) = 26.981541 \text{ amu}.$$

Ans. Neutron separation energy S_n of a nucleus $^{A}_{Z}X$ is given by

$$S_n = [m_N(^{A-1}_{Z}X) + m_n - m_N(^{A}_{Z}X)]\,c^2$$

$$= [\{m_N(^{A-1}_{Z}X) + Zm_e\} + m_n - \{m_N(^{A}_{Z}X) + Zm_e\}]\,c^2$$

$$= [m(^{A-1}_{Z}X) + m_n - m(^{A}_{Z}X)]\,c^2$$

Here m_N refers to nuclear mass while m refers to atomic mass.

$\therefore$ Neutron separation energy of $^{41}_{20}$Ca is given by

$$S_n(^{41}_{20}Ca) = [m(^{40}_{20}Ca) + m_n - m(^{41}_{20}Ca)]\,c^2$$

$$= [39.962591 + 1.008665 - 40.962278] \times 931 \text{ MeV}$$

[By using $c^2 = 931$ MeV / amu]

$$= 0.008978 \times 931 \text{ MeV}$$

$$= \mathbf{8.36\ MeV}$$

Neutron separation energy of $^{27}_{13}$Al is given by

$$S_n(^{27}_{13}Al) = [m(^{26}_{13}Al) + m_n - m(^{27}_{13}Al)]\,c^2$$

$$= [25.986895 + 1.008665 - 26.981541] \times 931 \text{ MeV}$$

$$= 0.014019 \times 931 \text{ MeV}$$

$$= \mathbf{13.05\ MeV}.$$

13.25. *A source contains two phosphorous radio nuclides* $^{32}_{15}P(T_{1/2} = 14.3\ d)$ *and* $^{33}_{15}P(T_{1/2} = 25.3\ d)$*. Initially, 10% of the decays come from* $^{33}_{15}P$*. How long one must wait until 90% do so ?*

Ans. Clearly, the source has initially 90% of $^{32}_{15}$P radionuclides and 10% of $^{33}_{15}$P radionuclides. Finally, say after t days, the source has 10% of $^{32}_{15}$P radionuclides and 90% of $^{33}_{15}$P radionuclides.

$\therefore$ Initial number of $^{32}_{15}$P nuclides = $9x$

Initial number of $^{33}_{15}$P nuclides = x

Final number of $^{32}_{15}$P nuclides = y

Final number of $^{33}_{15}$P nuclides = $9y$

As $$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}} = (2)^{-\frac{t}{T_{1/2}}}$$

or $$N = N_0\,(2)^{-\frac{t}{T_{1/2}}}$$

For first isotope,

$$y = 9x\,(2)^{-\frac{t}{14.3}}$$

For second isotope,

$$9y = x\,(2)^{-\frac{t}{25.3}}$$

On dividing, we get

$$9 = \frac{1}{9}(2)^{t\left(\frac{1}{14.3} - \frac{1}{25.3}\right)}$$

or $$81 = 2^{\frac{11t}{14.3 \times 25.3}}$$

or $$\log 81 = \frac{11t}{14.3 \times 25.3}\log 2$$

or $$1.9085 = \frac{11t \times 0.3010}{14.3 \times 25.3}$$

or $$t = \frac{1.9085 \times 14.3 \times 25.3}{11 \times 0.3010} = \mathbf{208.5\ days.}$$

13.26. *Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α particle. Consider the following decay process :*

$$^{223}_{88}Ra \longrightarrow {}^{209}_{82}Pb + {}^{14}_{6}C$$

$$^{223}_{88}Ra \longrightarrow {}^{219}_{86}Rn + {}^{4}_{2}He$$

Calculate the Q-values for these decays and determine that both are energetically allowed.

Ans. The Q–value for the first decay process is given by

$$Q = m(^{223}_{88}\text{Ra}) - m(^{209}_{82}\text{Pb}) - m(^{14}_{6}\text{C})$$
$$= [223.01850 - 208.981107 - 14.000324] \text{ amu} \times c^2$$
$$= 0.034109 \times 931.5 \text{ MeV} = \mathbf{31.85 \text{ MeV}}$$

For the second process,

$$Q = m(^{223}_{88}\text{Ra}) - m(^{219}_{86}\text{Rn}) - m(^{4}_{2}\text{He})$$
$$= [223.01850 - 219.00948 - 4.00260] \text{ amu} \times c^2$$
$$= 0.00642 \times 931.5 = \mathbf{5.98 \text{ MeV}}$$

As the Q-value is positive in both cases, so both decay processes are energetically possible.

13.27. *Consider the fission of $^{238}_{92}U$ by fast neutrons. In one fission event, no neutrons are emitted and the final stable end products, after the beta-decay of the primary fragments, are $^{140}_{58}Ce$ and $^{99}_{44}Ru$. Calculate Q for this fission process. The relevant atomic and particle masses are*

$$m(^{238}_{92}U) = 238.05079 \text{ amu}$$
$$m(^{140}_{58}Ce) = 139.90543 \text{ amu}$$
$$m(^{99}_{44}Ru) = 98.90594 \text{ amu}$$
$$m_n = 1.00867 \text{ amu}$$

Ans. The fission may be represented as

$$^{238}_{92}\text{U} + ^{1}_{1}n \longrightarrow ^{140}_{58}\text{Ce} + ^{99}_{44}\text{Ru} + 10\ ^{0}_{-1}e + Q$$

The Q-value for the process is

$$Q = [m_N(^{238}_{92}\text{U}) + m_n - m_N(^{140}_{58}\text{Ce}) - m_N(^{99}_{44}\text{Ru}) - 10\, m_e]\, c^2$$

In terms of atomic masses, we can write

$$Q = [\{m(^{238}_{92}\text{U}) - 92\, m_e\} + m_n - \{m(^{140}_{58}\text{Ce}) - 58\, m_e\} - \{m(^{99}_{44}\text{Ru}) - 44\, m_e\} - 10\, m_e]\, c^2$$
$$= [m(^{238}_{92}\text{U}) + m_n - m(^{140}_{58}\text{Ce}) - m(^{99}_{44}\text{Ru})]\, c^2$$
$$= [238.05079 + 1.00867 - 139.90543 - 98.90594] \text{ amu} \times c^2$$
$$= (239.05946 - 238.81137) \text{ amu} \times 931.5 \frac{\text{MeV}}{\text{amu}}$$
$$= 0.24809 \times 931.5 = 231.09 \text{ MeV}$$
$$= \mathbf{231.1 \text{ MeV}}.$$

13.28. *Consider the D-T reaction (deuterium-tritium fusion) given by the equation :*

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + n$$

(a) Calculate the energy released in MeV in this reaction from the data :

$$m(^{2}_{1}H) = 2.014102 \text{ amu}$$
$$m(^{3}_{1}H) = 3.016049 \text{ amu}$$
$$m(^{4}_{2}He) = 4.002603 \text{ amu}$$
$$m_n = 1.00867 \text{ amu}$$

[CBSE D 03C, 08C]

(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the Coulomb repulsion ? To what temperature must the gases be heated to initiate the reaction ?

Ans. (*a*) The net reaction is

$$^{2}_{1}\text{H} + ^{3}_{1}\text{H} \longrightarrow ^{4}_{2}\text{He} + ^{1}_{0}n + Q$$

∴ Energy released in the reaction is

$$Q = [m_N(^{2}_{1}\text{H}) + m_N(^{3}_{1}\text{H}) - m_N(^{4}_{2}\text{He}) - m_n]\, c^2$$

Adding and subtracting 2 m_e, we get

$$Q = [\{m_N(^{2}_{1}\text{H}) + m_e\} + \{m_N(^{3}_{1}\text{H} + m_e)\} - \{m_N(^{4}_{2}\text{He}) + 2m_e\} - m_n]\, c^2$$
$$Q = [m(^{2}_{1}\text{H}) + m(^{3}_{1}\text{H}) - m(^{4}_{2}\text{He}) - m_n]\, c^2$$
$$= [2.014102 + 3.016049 - 4.002603 - 1.00867] \times 931.5 \text{ MeV}$$
$$= 0.018883 \times 931 \text{ MeV} = \mathbf{17.58 \text{ MeV}}.$$

(*b*) Distance between the nuclei when they almost touch other is

$$d = 2r = 2 \times 1.5 \times 10^{-15} \text{ m} = 3 \times 10^{-15} \text{ m}$$

∴ Repulsive potential energy of the two nuclei when they almost touch other

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{d}$$
$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{3 \times 10^{-15}} \text{ J}$$
$$= 7.68 \times 10^{-14} \text{ J} \quad [\because\ q_1 = q_2 = 1.6 \times 10^{-19} \text{C}]$$

K.E. required for one fusion event

= Average thermal K.E. available with the interacting particles

$$= 2 \times \frac{3}{2} kT = 2kT$$

$$\therefore \quad 2kT = 7.68 \times 10^{-14} \text{ J}$$

or $$T = \frac{7.68 \times 10^{-14} \text{ J}}{2 \times 1.38 \times 10^{-23} \text{ JK}^{-1}} = \mathbf{1.85 \times 10^9 \text{ K}}.$$

13.29. *Obtain the maximum kinetic energy of β-particles and the radiation frequencies corresponding to γ-decays in the decay scheme shown in Fig. 13.21. You are given that,*

$$m(Au^{198}) = 197.968233 \text{ amu}$$
$$m(Hg^{198}) = 197.966760 \text{ amu}$$

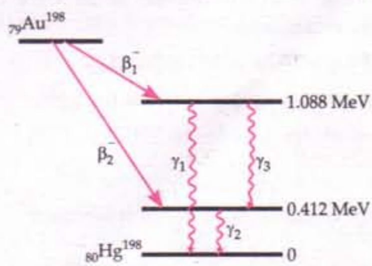

Fig. 13.21

Ans. The frequencies of γ-radiation will be equal to the corresponding energy differences divided by Planck's constant h.

$$\nu = \frac{E_2 - E_1}{h}$$

$$\therefore \nu(\gamma_1) = \frac{(1.088 - 0) \times 1.6 \times 10^{-13}}{6.63 \times 10^{-34}} = \mathbf{2.627 \times 10^{20}\ Hz}$$

$$\nu(\gamma_2) = \frac{(0.412 - 0) \times 1.6 \times 10^{-13}}{6.63 \times 10^{-34}} = \mathbf{9.949 \times 10^{19}\ Hz}$$

$$\nu(\gamma_3) = \frac{(1.088 - 0.412) \times 1.6 \times 10^{-13}}{6.63 \times 10^{-34}} = \mathbf{1.632 \times 10^{19}\ Hz}$$

The β_1^--decay can be represented as

$$^{198}_{79}\text{Au} \longrightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}e + Q(\beta_1^-) + Q(\gamma_1)$$

where $Q(\gamma_1) = 1.088$ MeV.

∴ Maximum kinetic energy of β_1^- particle is

$$K_{max}(\beta_1^-) = \left[m(^{198}_{79}\text{Au}) - \left\{ m(^{198}_{80}\text{Hg}) + \frac{1.088}{931.5} \right\} \right] c^2$$

[*Neglecting the rest mass of β-particle*]

$$= \left[197.968233 - \left(197.966760 + \frac{1.088}{931.5} \right) \right] \times 931.5 \text{ MeV}$$

[∵ $c^2 = 931.5$ MeV / amu]

$= (0.001473 \times 931.5 - 1.088)$ MeV

$= (1.372 - 1.088)$ MeV $= \mathbf{0.284\ MeV}$

The β_2^- decay can be represented as

$$^{198}_{79}\text{Au} \longrightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}e + Q(\beta_2^-) + Q(\gamma_2)$$

where $Q(\gamma_2) = 0.412$ MeV.

∴ Maximum kinetic energy of β_2^- particle is

$$K_{max}(\beta_2^-) = \left[m(^{198}_{79}\text{Au}) - \left\{ m(^{198}_{80}\text{Hg}) + \frac{0.412}{931.5} \right\} \right] c^2$$

$$= \left[197.068233 - \left(197.966760 + \frac{0.412}{931.5} \right) \right] \times 931.5 \text{ MeV}$$

$= [0.001473 \times 931.5 - 0.412]$ MeV

$= [1.372 - 0.412]$ MeV $= \mathbf{0.960\ MeV.}$

13.30. *Calculate and compare the energy released by (a) fusion of 1.0 kg of hydrogen deep within the Sun and (b) the fission of 1.0 kg of ^{235}U in a fission reactor.*

Ans. (*a*) In the sun, 4 hydrogen nuclei combine to form a helium nucleus with the release of 26 MeV of energy. The net fusion reaction is

$$4\,{}^{1}_{1}\text{H} \longrightarrow {}^{4}_{2}\text{He} + 2e^+ + 26 \text{ MeV}$$

Number of atoms in 1 g of $^{1}_{1}$H $= 6 \times 10^{23}$

Number of atoms in 1 kg or 1000 g of $^{1}_{1}$H

$= 6 \times 10^{23} \times 1000 = 6 \times 10^{26}$

The energy released by 1 kg of hydrogen

$$= \frac{26 \times 6 \times 10^{26}}{4} = \mathbf{39 \times 10^{26}\ MeV.}$$

(*b*) Number of atoms in 235 g of ^{235}U $= 6 \times 10^{23}$

Number of atoms in 1 kg or 1000 g of ^{235}U

$$= \frac{6 \times 10^{23} \times 1000}{235}$$

Energy released in per fission of ^{235}U = 200 MeV

Energy released in fission of 1 kg of ^{235}U

$$= \frac{6 \times 10^{26} \times 200}{235} = \mathbf{5.1 \times 10^{26}\ MeV.}$$

Thus the energy released in fusion of 1 kg of hydrogen fusion is about 8 times that of energy released in fission of 1 kg ^{235}U.

13.31. *Suppose India has a target of producing by 2020 A.D., 200,000 MW of electric power, 10 percent of which is to be obtained from nuclear power plants. Suppose we are given that, on average the efficiency of utilisation (i.e., conversion to electric energy) of thermal energy produced in a reactor is 25%. How much amount of fissionable uranium would our country need per year at the turn of this century ? Take the heat energy per fission of U^{235} to be about 200 MeV. Avogadro's Number $N = 6.023 \times 10^{23}\ mol^{-1}$.*

Ans. Target of power by 2020 A.D.

$= 2 \times 10^5$ MW $= 2 \times 10^{11}$ W

Power required from nuclear power plants

$= 10\%$ of 2×10^{11} W

$$= \frac{10}{100} \times 2 \times 10^{11} = 2 \times 10^{10} \text{ W}$$

∴ Energy required from nuclear power plants per year

$=$ Power × time

$= 2 \times 10^{10} \times 365.25 \times 24 \times 60 \times 60$ J

$= 6.312 \times 10^{17}$ J

Energy released per fission = 200 MeV

Available electrical energy per fission of ^{235}U nucleus

$= 25\%$ of 200 MeV

$$= \frac{25}{100} \times 200 \text{ MeV} = 25 \times 2 \times 1.6 \times 10^{-13} \text{J}$$

$= 8 \times 10^{-12}$ J [∵ 1 MeV $= 1.6 \times 10^{-13}$ J]

Number of ^{235}U fissions required per year

$$= \frac{6.312 \times 10^{17}}{8 \times 10^{-12}} = 7.89 \times 10^{28}$$

Required number of moles of ^{235}U

$$= \frac{7.89 \times 10^{28}}{\text{Avogadro's number}} = \frac{7.89 \times 10^{28}}{6.023 \times 10^{23}}$$

$= 13.1 \times 10^4$

Mass of ^{235}U required

$=$ Number of moles × mass number

$= 13.1 \times 10^4 \times 235$ g

$= 3078.5 \times 10^4$ g $\simeq \mathbf{3.078 \times 10^4\ kg.}$

Text Based Exercises

TYPE A : VERY SHORT ANSWER QUESTIONS (1 mark each)

1. How many electrons, protons and neutrons are there in a nucleus of atomic number 11 and mass number 24 ? [IIT 02]
2. Write the number of protons and neutrons in $^{144}_{56}Ba$. [CBSE D 93C]
3. What are the number of protons and the number of neutrons in a nucleus of $^{238}_{92}U$?
4. What is meant by the term isotope ? [ISCE 98 ; Punjab 99]
5. What are isotones ? [Punjab 99, 02]
6. What are isomeric nuclides ?
7. Give the approximate magnitude of the radius of a nucleus.
8. What is the ratio of volume of atom to the volume of nucleus ?
9. Is the rest mass of a proton exactly equal to or nearly equal to the rest mass of a neutron ?
10. Write the names and formulae of three isotopes of hydrogen.
11. Select the pairs of isotopes and isotones from the following nuclei :

$$^{13}_{6}C,\ ^{14}_{7}C,\ ^{30}_{15}P,\ ^{31}_{15}P$$

12. Select the pairs of isobars and isotones from the following nuclei :

$$^{14}_{6}C,\ ^{13}_{7}N,\ ^{14}_{7}N,\ ^{16}_{8}O$$

13. Separate out the isoneutronic nuclei from the following :

$$^{3}_{1}H,\ ^{4}_{2}He,\ ^{23}_{11}Na,\ ^{24}_{12}Mg$$

14. State Einstein's mass-energy relation. [ISCE 97]
15. Define atomic mass unit. [ISCE 95 ; Haryana 97C]
16. Express one atomic mass unit in kilogram. [Haryana 94]
17. How many joule are there in 1 MeV ? [Himachal 97]
18. State the energy equivalent of 1 amu in MeV.
19. How many electron volts make up one joule ? [Haryana 93]
20. What is the energy equivalent of the mass of an electron ?
21. Express the mass of electron and proton in amu.
22. Write the relation connecting the radius R, of nucleus and its mass number A. [CBSE OD 13C]
23. What is the order of magnitude of nuclear mass density ? [Punjab 97]
24. How does the nuclear mass density depend on the size of the nucleus ?
25. Is the nuclear density same for all elements ?
26. In heavy nuclei, is the ratio of the number of protons and the number of neutrons less than, greater than or equal to unity ?
27. Name the largest stable nucleus.
28. What is the ratio of kWh to MeV ?
29. Who discovered neutrons ?
30. How many times is the nuclear force stronger than the electrostatic force ?
31. Define the term 'mass defect' of a nucleus. How is it related with its binding energy ? [CBSE OD 14C]
32. Define binding energy of a nucleus.
33. A nucleus, of mass number A has a mass defect Δm. Give the formula, for the binding energy per nucleon, of this nucleus. [CBSE D 04C]
34. What do you mean by the fact that binding energy of helium nucleus is 28.17 MeV ?
35. What do you mean by binding energy per nucleon ?
36. What is the relation between the binding energy per nucleon and stability of a nucleus ?
37. What is the order of binding energy per nucleon for most of the nuclei ?
38. Name three nuclei which lie on maxima of binding energy curve.
39. Name three nuclei which lie on minima of binding energy curve.
40. State two characteristic properties of nuclear forces. [CBSE D 08 ; OD 11]
41. Give an equation representing the decay of a neutron. [Punjab 98]
42. Which is unstable among electron, proton, neutron and α-particle ?
43. Identify the nuclides X and Y in the nuclear reactions : [ISCE 01]

$$^{11}_{5}B + ^{1}_{1}H \longrightarrow ^{8}_{4}Be + X\ ; \quad ^{14}_{6}C \longrightarrow Y + ^{0}_{-1}e$$

44. Define the term radioactivity.
45. Write the radioactive radiations in the order of increasing ionising power.
46. Write the radioactive radiations in the order of increasing penetrating power.
47. Name the scientist who discovered radioactivity.
48. Name the two radioactive elements discovered by Curie couple.

49. What are radioactive substances ?

50. Which have greater ionising power : alpha particles or beta particles ? [CBSE D 96]

51. After losing two electrons, an atom of helium becomes equivalent to which particle : α, β or γ.

52. Among alpha, beta and gamma radiations, which are the ones affected by the magnetic field ? [CBSE 04 ; ISCE 96]

53. Among alpha, beta and gamma radiations, which get deflected by the electric field ? [CBSE OD 04]

54. How will you define range of α-particles ?

55. Write the nuclear decay process for β-decay of $^{32}_{15}P$. [CBSE D 04 ; OD 12]

56. Draw the graph showing the distribution of kinetic energy of electrons emitted during beta-decay. [CBSE D 06C]

57. Does the ratio of neutrons to protons in a nucleus increase, decrease or remain the same after the emission of an α-particle ? [CBSE D 1994]

58. Define half life of a radioactive material. [CBSE OD 02 ; F 09 ; Haryana 04]

59. Define radioactive decay constant. [Punjab 99C]

60. Write the relation between decay constant λ and half life $T_{1/2}$ of a radioactive element. [CBSE OD 03C]

61. Define average life of a radioactive substance. [Punjab 99C ; ISCE 01]

62. A radioactive nuclide has a decay constant equal to λ. Give the formula for the (*i*) half life and (*ii*) mean life of this nuclide. [CBSE OD 03C, 07C]

63. Write a relation between half life $T_{1/2}$ and average life τ of a radioactive substance. [CBSE OD 14C]

64. The mean life of a radioactive sample is T_m. What is the time in which 50% of this sample would get decayed ? [CBSE Sample Paper 11]

65. How is the mean life of a radioactive sample related to its half-life ? [CBSE F 11]

66. What is meant by the activity of a radioactive sample ? [ISCE 97 ; CBSE OD 14C]

67. How does the activity of a radioactive sample vary with time ?

68. A radioactive sample is dissolved in a liquid and the solution is heated. Will the activity of the solution be same as that of the sample ?

69. Define a curie.

70. Define one rutherford. How it is related with curie ?

71. Write the value of 1 millicurie in terms of rutherford.

72. Write the SI unit for the activity of a radioactive nuclide. [CBSE OD 06C, 14C]

73. Carbon-14 ($^{14}_{6}C$) is an isotope of carbon. It is radioactive, decaying to nitrogen-14 ($^{14}_{7}N$). Write the decay equation. [CBSE F 95]

74. An atomic nucleus denoted by $^{A}_{Z}X$ emits an alpha particle. Write an equation to show the formation of the daughter product. [ISCE 99]

75. A nucleus $^{235}_{92}U$ undergoes alpha decay and transforms to thorium. What is (*i*) the mass number and (*ii*) atomic number of the nucleus produced ? [ICSE 97 ; CBSE D 11C]

76. A $^{235}_{92}U$ nucleus emits two α-particles and two β-particles and transforms into a thorium nucleus. What is the mass number and atomic number of the thorium nucleus so produced ? [CBSE F 10]

77. $^{238}_{92}U$ on absorbing a neutron goes over to $^{239}_{92}U$. This nucleus emits an electron to go over to neptunium which on further emitting an electron goes to plutonium. How would you represent the resulting plutonium ?

78. A radioactive nucleus undergoes a series of decays according to the sequence :

$$A \xrightarrow{\beta} A_1 \xrightarrow{\alpha} A_2 \xrightarrow{\alpha} A_3$$

If the mass number and atomic number of A_3 are 172 and 69 respectively, what are the mass number and atomic number of A ? [CBSE OD 94]

79. Write the names of the four radioactive series.

80. Give the electric charges of positron and photon in units of electric charge.

81. A radioactive isotope of silver has half-life of 20 minutes. What fraction of the original mass would remain after one hour ? [ISCE 93]

82. A radioactive substance has a half-life period of 30 days. What is the disintegration constant ? [Himachal 97]

83. The half-life period of a radioactive substance is 30 days. What is the time taken for 3/4th of its original mass to disintegrate ? [CBSE OD 94]

84. What is a nuclear reaction ?

85. Name the quantities that are conserved in nuclear reaction.

86. What is nuclear energy ?

87. What do you mean by *Q*-value of a nuclear reaction ? [Punjab 2000, 02, 04]

88. What is a fissile material ?

89. Out of $^{235}_{92}U$ and $^{238}_{92}U$ isotopes, which is fissile and which one is fertile ?

90. What are thermal neutrons ? [Himachal 03]

91. What is the role of moderator in nuclear reactor ? [ISCE 98]

92. What is the function of a moderator in a nuclear reactor ? Name any one substance which is commonly used as a moderator. [ISCE 92]

93. What is the function of heavy water in a nuclear reactor ? [ISCE 02]
94. What is meant by a self-sustained nuclear reaction ?
95. What is the role of control rods in nuclear reactor ? [ISCE 98]
96. What is the principle of operation of a nuclear reactor ? [ISCE 02]
97. What is critical size and critical mass as regards to nuclear fission ?
98. How much is the critical mass for uranium fuel ?
99. How much energy is released in per fission of $^{235}_{92}U$?
100. What are the essential parts of a nuclear reactor ?
101. Write any one equation representing nuclear fusion reaction. [CBSE OD 96]
102. Name the reaction responsible for energy production in the sun. [Himachal 98 ; CBSE 04]

Or

Name the principal source of sun's energy. [ISCE 93]
103. Name the phenomenon by which energy is produced in a star. [Punjab 97C]
104. Write the β-decay of tritium in symbolic form. [CBSE F 15]
105. A certain radioactive element disintegrates for a time interval equal to its mean life
   (*i*) what fraction of the element remains undecayed ?
   (*ii*) what fraction has decayed ?
106. Assuming the nuclei to be spherical in shape, how does the surface area of a nucleus of mass number A_1 compare with that of a nucleus of mass number A_2 ? [CBSE OD 08 C]
107. What is the nuclear radius of ^{125}Fe, if that of ^{27}Al is 3.6 fermi ? [CBSE OD 08]
108. A nucleus undergoes β-decay. How does its (*i*) mass number (*ii*) atomic number change ? [CBSE D 11C]
109. Four nuclei of an element fuse together to form a heavier nucleus. If the process is accompanied by release of energy, which of the two-the parent or the daughter nucleus would have a higher binding energy/nucleon ? [CBSE Sample Paper 08]
110. Name the absorbing material used to control the reaction rate of neutrons in a nuclear reactor. [CBSE D 08]
111. Two nuclei have mass numbers in the ratio 2 : 5. What is the ratio of their nuclear densities ? [CBSE D 09]
112. Two nuclei have mass numbers in the ratio 27 : 125. What is the ratio of their nuclear radii ? [CBSE D 09]
113. Plot a graph showing the variation of potential energy of a pair of nucleons as a function of their separation. [CBSE OD 09]
114. Write the basic nuclear process underlying β-decay of a given radioactive nucleus. [CBSE D 13C]

Answers

1. Number of protons or electrons = Z = **11**.
   Number of neutrons = $A - Z = 24 - 11 = \mathbf{13}$.
2. Number of protons = Z = **56**.
   Number of neutrons = $A - Z = 144 - 56 = \mathbf{88}$.
3. Number of protons = Z = **92**.
   Number of neutrons = $A - Z = 238 - 92 = \mathbf{146}$.
4. Refer to point 6 of Glimpses on page 13.77.
5. Refer to point 8 of Glimpses on page 13.77.
6. Refer to point 9 of Glimpses on page 13.77.
7. 10^{-14} m.
8. 10^{15}.
9. Nearly equal to. In fact, the rest mass of a neutron is sightly more than that of a proton.
10. Hydrogen (^1_1H), deuterium (^2_1H) and tritium (^3_1H)
11. (*i*) ($^{30}_{15}P$ and $^{31}_{15}P$) are isotopes.
   (*ii*) $^{13}_6C$ and $^{14}_7C$ are isotones.
12. (*i*) $^{14}_6C$ and $^{14}_7N$ are isobars.
   (*ii*) $^{14}_6C$ and $^{16}_8O$ are isotones.
13. The isoneutronic pairs are
   (*i*) ^3_1H and ^4_2He and (*ii*) $^{23}_{11}Na$ and $^{24}_{12}Mg$.
14. When *m* mass is converted into energy in any process, the energy obtained is given by Einstein's mass-energy relation, $E = mc^2$.
15. 1 atomic mass unit = $\frac{1}{12}$th of the mass of $^{12}_6C$ atom.
16. $1\,\text{amu} = 1.66 \times 10^{-27}$ kg.
17. $1\,\text{MeV} = 10^6\,\text{eV} = 10^6 \times 1.6 \times 10^{-19}\,\text{J} = 1.6 \times 10^{-13}$ J.
18. 1 amu = 931 MeV.
19. $6.242 \times 10^{-19}\,\text{eV} = 1\text{J}$
20. $m_e = 0.511\,\text{eV}$
21. $m_e = 0.00055$ amu, $m_p = 1.007825$ amu.
22. Nuclear radius, $R = R_0 A^{1/3}$
   where $R_0 = 1.1 \times 10^{-15}$ m.
23. $10^{17}\,\text{kgm}^{-3}$.
24. Nuclear mass density is independent of the size of the nucleus.

25. Yes.
26. Less than unity.
27. Lead (Pb).
28. $\dfrac{1\text{ kWh}}{1\text{ MeV}} = \dfrac{3.6\times10^{6}\text{ J}}{1.6\times10^{-13}\text{ J}} = \mathbf{2.25\times10^{19}}$.
29. James Chadwick in 1932.
30. 100 times.
31. Refer to point 16 of Glimpses on page 13.78.
$$\text{B.E.} = \Delta m \times c^2$$
32. Refer to point 17 of Glimpses on page 13.78.
33. B.E. per nucleon $= \dfrac{\Delta m \times c^2}{A}$
where c is the speed of light.
34. This means that 28.17 MeV energy is required to separate the 2 neutrons and 2 protons of helium nucleus to an infinite distance apart.
35. Refer to point 18 of Glimpses on page 13.78.
36. Larger the binding energy per nucleon, more stable is the nucleus.
37. 8 MeV per nucleon.
38. The nuclei which lie on maxima are ${}_2^4\text{He}$, ${}_6^{12}\text{C}$ and ${}_8^{16}\text{O}$.
39. The nuclei which lie on minima of binding energy curve are ${}_1^2\text{H}$, ${}_3^6\text{Li}$ and ${}_5^{10}\text{B}$.
40. (*i*) Nuclear forces are the strongest forces in nature.
(*ii*) These are short range forces.
41. ${}_0^1n \longrightarrow {}_1^1\text{H} + {}_{-1}^{0}e + \bar{\nu}$.
42. Neutron is unstable.
43. $X = {}_2^4\text{He}$ and $Y = {}_7^{14}\text{N}$.
44. Refer to point 21 of Glimpses on page 13.78.
45. γ-rays, β-rays and α-rays.
46. α-rays, β-rays and γ-rays.
47. Henry Becquerel.
48. Radium and polonium.
49. Refer to point 21 of Glimpses on page 13.78.
50. Alpha particles.
51. Alpha particle.
52. Alpha and beta radiations are affected by the magnetic field.
53. Alpha and beta radiations are deflected by the electric field.
54. The distance travelled by α-particles through air at S.T.P. before they lose their ionising power is called range of α-particles.
55. ${}_{15}^{32}\text{P} \rightarrow {}_{16}^{32}\text{S} + {}_{-1}^{0}e + \bar{\nu}$.
56. See Fig. 13.11 on page 13.21.
57. As $\dfrac{N-2}{Z-2} > \dfrac{N}{Z}$
i.e., Neutrons to protons ratio after α-decay > Neutrons to protons ratio before α-decay.
i.e., Neutrons to protons ratio increases after α-decay.
58. Refer to point 28 of Glimpses on page 13.79.
59. Refer to point 27 of Glimpses on page 13.79.
60. $T_{1/2} = \dfrac{0.693}{\lambda}$.
61. Refer to point 29 of Glimpses on page 13.79.
62. (i) Half life, $T_{1/2} = \dfrac{0.693}{\lambda}$. (ii) Average life, $\tau = \dfrac{1}{\lambda}$.
63. $T_{1/2} = 0.693\,\tau$ or $\tau = 1.44\,T_{1/2}$.
64. Time needed = Half life = $0.693\,T_m$
65. $\tau = 1.44\,T_{1/2}$
66., 67. The activity of a sample is defined as the number of radioactive disintegrations taking place per second in the sample.
It can be expressed as
$$R = \left|\frac{dN}{dt}\right| = \lambda N = \lambda N_0 e^{-\lambda t}$$
As N decreases exponentially with time, R also decreases exponentially with time.
68. The activity of the sample will remain same even in the solution state.
69. One curie is the decay rate of 3.7×10^{10} disintegrations per second.
70. One rutherford is the decay rate of 10^6 disintegrations per second.
$$1 \text{ curie} = 3.7\times10^{4} \text{ rutherford.}$$
71. 1 mCi (millicurie) = 37 rd (rutherford).
72. The SI unit for activity is becquerel.
1 becquerel = 1 Bq = 1 decay per second.
73. ${}_6^{14}\text{C} \longrightarrow {}_7^{14}\text{N} + {}_{-1}^{0}e + \bar{\nu}$.
74. ${}_Z^A\text{X} \longrightarrow {}_{Z-2}^{A-4}\text{Y} + {}_2^4\text{He}$.
75. ${}_{92}^{238}\text{U} \longrightarrow {}_{90}^{234}\text{Th} + {}_2^4\text{He} + \text{Q}$
(*i*) Mass number = **234**.
(*ii*) Atomic number = **90**.
76. ${}_{92}^{238}\text{U} \longrightarrow {}_{90}^{230}\text{Th} + 2\,{}_2^4\text{He} + 2\,{}_{-1}^{0}e$
(*i*) Mass number = **230**.
(*ii*) Atomic number = **90**.
77. ${}_{92}^{238}\text{U} \xrightarrow{+{}_0^1n} {}_{92}^{239}\text{U} \xrightarrow{-{}_{-1}^{0}e} {}_{93}^{239}\text{Np} \xrightarrow{-{}_{-1}^{0}e} {}_{94}^{239}\text{Pu}$
The resulting plutonium is ${}_{94}^{239}\text{Pu}$

78. $^{180}_{72}A \xrightarrow[\ ^{0}_{-1}e]{\beta} {}^{180}_{73}A_1 \xrightarrow[\ ^4_2He]{\alpha} {}^{176}_{71}A_2 \xrightarrow[\ ^4_2He]{\alpha} {}^{172}_{69}A_3$

$\therefore$ Mass number of A = 180
Atomic number of A = 72

79. (*i*) Uranium series (*ii*) Actinium series (*iii*) Thorium series and (*iv*) Neptunium series.

80. Charge on a positron = $+1.6 \times 10^{-19}$C.
Charge on photon = 0.

81. Number of half-lives,

$$n = \frac{\text{Total time of disintegration}}{\text{Half - life}} = \frac{60 \text{ min}}{20 \text{ min}} = 3$$

$$\therefore \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

82. $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{30} = \mathbf{0.0231\ day^{-1}}$.

83. Here $\frac{N_0 - N}{N_0} = \frac{3}{4}$

or $\frac{N}{N_0} = \frac{1}{4} = \left(\frac{1}{2}\right)^2$

$\therefore$ No. of half-lives, $n = 2$

Time of disintegration
= Half-life × No. of half-lives
= 30 × 2 = **60 days**

84. Refer to point 36 of Glimpses on page 13.79.

85. (*i*) Momentum (*ii*) Number of nucleons (*iii*) Charge and (*iv*) Energy.

86. Nuclear energy is the energy released in a nuclear reaction.

87. The amount of energy released or absorbed in a nuclear reaction is called its *Q*-value.

$$\underset{\text{Nucleus}}{A} + \underset{\text{Elementary particle}}{a} \longrightarrow \underset{\text{Nucleus}}{B} + \underset{\text{Elementary particle}}{b} + \underset{\text{Energy}}{Q}$$

88. A material which can undergo nuclear fission easily is called a fissile material.

89. $^{235}_{92}U$ is fissile while $^{238}_{92}U$ is fertile i.e. it can be made fissionable.

90. Slow moving neutrons of energy 0.0253 eV and having velocities of about 2200 ms^{-1} are called thermal neutrons.

91. A moderator slows down fast moving neutrons to thermal velocities so that they can cause fission of $^{235}_{92}U$ nuclei *e.g.*, heavy water, graphite, beryllium, etc.

92. Refer answer to the above Q. 91.

93. Heavy water is used as a moderator *i.e.*, it slows down the fast moving neutrons.

94. If the neutrons released in a nuclear fission can be used to promote further fission, then the reaction is a self-sustained nuclear reaction.

95. Control rods are used to start, stop or adjust a nuclear fission at a steady rate. By using control rods, the average number of neutrons per fission can be made one.

96. A nuclear reactor uses controlled chain reaction to produce energy. Here fast neutrons are slowed down by elastic scattering with moderators like heavy water. The reaction rate is controlled by neutron absorbing materials like cadmium rods.

97. The size of the fissionable material for which the multiplication factor $k = 1$ is called critical size and its mass is called critical mass. Such a mass maintains the chain reaction steady or sustained.

98. 10 kg.

99. About 200 MeV.

100. The essential parts of a nuclear reactor are :
(*i*) nuclear fuel (*ii*) moderator (*iii*) control rods
(*iv*) coolant (*v*) thick shielding.

101. $^2_1H + {}^2_1H \longrightarrow {}^4_2He + 24$ MeV

102. Fusion of hydrogen nuclei into helium nucleus.

103. Nuclear fusion.

104. $^3_1H \rightarrow {}^3_2He + {}^{0}_{-1}e + \bar{\nu} + Q$

105. (*i*) 0.368 (*ii*) 0.632

106. $\frac{S_1}{S_2} = \left(\frac{R_1}{R_2}\right)^2 = \left[\left(\frac{A_1}{A_2}\right)^{1/3}\right]^2 = \left(\frac{A_1}{A_2}\right)^{2/3}.$

107. $\frac{R_{Fe}}{R_{Al}} = \left(\frac{125}{27}\right)^{1/3} = \frac{5}{3},$

$R_{Fe} = \frac{5}{3} R_{Al} = \frac{5}{3} \times 3.6 = \mathbf{6.0\ fermi}.$

108. (*i*) Mass number remains unchanged.
(*ii*) Atomic number increases by one unit.

109. The daughter nucleus would have a higher binding energy/nucleon.

110. Cadmium.

111. Ratio of nuclear density = 1 : 1, because nuclear density is independent of mass number.

112. $\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{27}{125}\right)^{1/3} = \frac{3}{5} = \mathbf{3:5}.$

113. See Fig. 13.4 on page 13.7.

114. $^A_ZX \rightarrow {}^{A}_{Z+1}Y + \beta^- + \bar{\nu}$

TYPE B : SHORT ANSWER QUESTIONS (2 or 3 marks each)

1. Define the terms : nucleons, atomic number, mass number, nuclear mass and nuclide, in relation to a nucleus.
2. Distinguish between isotopes, isobars and isotones with suitable examples. [CBSE OD 08 ; Punjab 11]
3. Show diagrammatically the isotopes of hydrogen. Why do they show similar chemical properties ?
4. Define atomic mass unit and electron volt. Derive relation between them. [Pubjab 98C, 01]
5. From the relation $R = R_0 A^{1/3}$, where R_0 is constant and A is the mass number of the nucleus, show that nuclear matter density is independent of A. [CBSE D 13, 15]
6. Describe how Chadwick discovered neutrons. Is neutron a stable particle when isolated ? [ISCE 95]
7. State four important properties of neutrons.
8. How are protons which are positively charged, held together inside a nucleus ? Draw a graph between potential energy of a pair of nucleons as a function of their separation.
   What is significance of negative potential energy in this graph ? [CBSE D 94C, F 06]
9. What are nuclear forces ? Give their important properties. [CBSE D 92 ; Punjab 01, 02]
10. (*i*) What characteristic property of nuclear force explains the constancy of binding energy per nucleon (*BE*/*A*) in the range of mass number '*A*' lying $30 < A < 170$?
    (*ii*) Show that the density of nucleus over a wide range of nuclei is constant, independent of mass number *A*. [CBSE D 12]
11. (*a*) Draw a graph showing the variation of potential energy of a pair of nucleons as a function of their separation. Indicate the regions in which nuclear force is (*i*) attractive, and (*ii*) repulsive.
    (*b*) Write three characteristic features of nuclear force which distinguish it from the coulomb force. [CBSE D 05 ; OD 10, 12]
12. What is mass defect of a nucleus ? Express it mathematically. How do you account for it ?
13. What is packing fraction ? Give its physical significance in relation to nuclear stability.
14. Draw a plot showing the variation of binding energy per nucleon with mass number *A*. Write two important conclusions which you can draw from this plot. Explain with the help of this plot, the release in energy in the processes of nuclear fusion and fission. [ISCE 01 ; CBSE F 05 ; D 06 ; OD 09, 11]
15. Draw a diagram to show the variation of binding energy per nucleon with mass number for different nuclei. State with reason why light nuclei usually undergo nuclear fusion. [CBSE OD 01, 06C]
16. Draw the graph showing the variation of binding energy per nucleon with mass number. Give the reason for the decrease of binding energy per nucleon for nuclei with high mass numbers. [CBSE D 04, OD 06C]
17. Draw a plot of the binding energy per nucleon as a function of mass number for a large number of nuclei. Explain the energy release in the process of nuclear fission from the above plot. Write a typical nuclear reaction in which a large amount of energy is released in the process of nuclear fission. [CBSE D 07C ; F 08 ; OD 08]
18. Draw a graph showing the variation of binding energy per nucleon with mass number of different nuclei. Mark the region where the nuclei are (*a*) most stable (*b*) prone to fusion and (*ii*) prone to fission. [CBSE D 96]
19. (*a*) Write the relation for binding energy (BE) (in MeV) of a nucleus of mass ($m\,{}^{A}_{Z}X$), atomic number (Z) and mass number (A) in terms of the masses of its constituents – neutrons and protons.
    (*b*) Draw a plot of BE/A versus mass number A for $2 \le A \le 170$. Use this graph to explain the release of energy in the process of nuclear fusion of two light nuclei. [CBSE D 14C]
20. State Sody-Fajan's displacement laws of radioactive transformation.
21. What is alpha decay ? Give an example.
22. Explain beta decay and given one example of this decay. [CBSE OD 94C ; Haryana 01]
23. Explain the emission of γ-rays with an example.
24. Define half life and mean life of a radiactive substance. What is the relation between the two ? [CBSE F 93]
25. Define disintegration constant and mean life of a radioactive substance. [CBSE OD 97]
26. Define the terms decay constant and half-life of a radioactive sample. Write their SI units. Derive the relation connecting the two. [Pubjab 01 ; CBSE D 01 OD 04, 06]
27. How will you establish experimentally that the radiation from a radioactive source consists of three distinct components ?

28. Derive the relation : $N = N_o e^{-\lambda t}$ for radioactive decay. [ISCE 03]

29. What do you mean by half-life and mean life of a radioactive substance ? Deduce the relation between them. [Haryana 99C]

30. Derive the expression for the law of radioactive decay of a given sample having initially N_0 nuclei decaying to the number N present at any subsequent time t. Plot a graph showing the variation of the number of nuclei versus the time t elapsed. Mark a point on the plot in terms of $T_{1/2}$ value when the number present $N = N_0/16$. [CBSE OD 08 ; F13]

31. A radionuclide sample has N_0 nuclei at $t = 0$. Its number of undecayed nuclei get reduced to N_0 / e at $t = \tau$. What does the term 'τ' stand for ? Write, in terms of 'τ', the time interval 'T' in which half of the original number of nuclei, of the radionuclide would have got decayed. [CBSE D 08C]

32. State the law of radioactive decay. Plot a graph showing the number (N) of undecayed nuclei as a function of time (t) for a given radioactive sample having half life $T_{1/2}$.

Depict in the plot the number of undecayed nuclei at (*i*) $t = 3T_{1/2}$ and (*ii*) $t = 5T_{1/2}$. [CBSE D 11]

33. (*a*) Define the activity of a radioactive nucleus and state its SI unit. [CBSE OD 14C]

(*b*) Two radioactive nuclei X and Y initially contain equal number of atoms. The half life is 1 hour and 2 hours respectively. Calculate the ratio of their rates of disintegration after two hours. [CBSE F 05]

34. Define the term decay constant of radioactive nucleus. Two nuclei P, Q have equal number of atoms at $t = 0$. Their half-lives are 3 hours and 9 hours respectively. Compare their rates of disintegration after 18 hours from the start. [CBSE D 06C]

35. (*a*) Draw the energy level diagram showing the emission of β-particles followed by γ-rays by a $^{60}_{27}Co$ nucleus.

(*b*) Plot the distribution of kinetic energy of β-particles and state why the energy spectrum is continuous. [CBSE OD 05]

36. (*a*) Deduce the expression, $N = N_0 e^{-\lambda t}$, for the law of radioactive decay.

(*b*) (*i*) Write symbolically the process expressing the β^+ decay of $^{22}_{11}Na$. Also write the basic nuclear process underlying this decay. (*ii*) Is the nucleus formed in the decay of the nucleus $^{22}_{11}Na$, an isotope or isobar ? [CBSE D 14]

37. Explain how the speed of α-particles emitted during a radioactive disintegration be estimated ?

38. Why is the energy distribution of β-rays continuous ? How did it lead to the discovery of neutrino ? [CBSE OD 93]

39. Explain the phenomenon of nuclear fission. Give one representative equation.

40. Explain how is fission an enormous source of energy. Or show that the release of energy per nuclear fission is about 200 MeV.

41. What is nuclear fission ? Explain how a chain reaction can occur in a fissionable material ? [CBSE F 94]

42. Define multiplication factor and critical size for a fissionable material. Give their significance.

43. What is a moderator ? Explain its action in slowing down the neutrons.

44. What is nuclear fusion ? Give one representative equation. [CBSE OD 92]

45. What is the main difference between fission reaction and fusion reaction ? Give one example of each. [ISCE 99]

46. State the necessary conditions for nuclear fusion to occur.

47. A nuclear bomb and a nuclear reactor work on the same principle. Explain why in one case explosion occurs and in the other energy is available at a steady rate.

48. Explain the source of energy in the sun. [Punjab 01, 02, 03]

49. What is nuclear holocaust ? [Punjab 05]

50. A radioactive nucleus 'A' undergoes a series of decays according to the following scheme :

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

The mass number and atomic number of A are 180 and 72 respectively. What are these numbers for A_4 ? [CBSE D 09]

51. (*a*) In a typical nuclear reaction, *e.g.*,

$$^2_1H + ^2_1H \longrightarrow ^3_2He + n + 3.27\,MeV,$$

although number of nucleons is conserved, yet energy is released. How ? Explain.

(*b*) Draw a plot of potential energy between a pair of nucleons as a function of their separation. Mark the regions where potential energy is (*i*) positive and (*ii*) negative. [CBSE D 13]

52. Complete the following nuclear reactions :

(*a*) $^{10}_5B + ^1_0n \longrightarrow ^4_2He + ...$

(*b*) $^{94}_{42}Mo + ^2_1H \longrightarrow ^{95}_{43}Te + ...$

[CBSE D 15C]

53. Complete the following nuclear reactions :

(a) (i) ${}^{208}_{84}Po \longrightarrow {}^{204}_{82}Pb + ...$

(ii) ${}^{32}_{15}P \longrightarrow {}^{32}_{16}S + ...$

(b) Write the basic process involved in nuclei responsible for (i) β^- and (ii) β^- decay. [CBSE OD 15C]

54. Write symbolically the nuclear β^+ decay process of ${}^{11}_{6}C$. Is the decayed product X an isotope or isobar of ${}^{11}_{6}C$? Given the mass values $m({}^{11}_{6}C) = 11.011434$ u and $m(X) = 11.009305$ u. Estimate the Q-value in this process. [CBSE OD 15]

55. Distinguish between nuclear fission and fusion. Show how in both these processes energy is released. Calculate the energy release in MeV in the deuterium-tritium fusion reaction :

$${}^{2}_{1}H + {}^{3}_{1}H \longrightarrow {}^{4}_{2}He + n$$

Using the data :

$m({}^{2}_{1}H) = 2.014102$ u ; $m({}^{3}_{1}H) = 3.016049$ u ;

$m({}^{4}_{2}H) = 4.002603$ u ;

$m_n = 1.008665$ u,

$1 \text{ u} = 931.5 \text{ MeV/c}^2$ [CBSE D 15]

Answers

1. Refer answer to Q. 2 on page 13.1.
2. Refer answer to Q. 3 on page 13.2.
3. Refer answer to Q. 3 on page 13.2.
4. Refer answer to Q. 4 on page 13.3.
5. Refer answer to Q. 6 on page 13.4.
6. Refer answer to Q. 7 on page 13.6.
7. Refer answer to Q. 7 on page 13.6.
8. Due to very strong attractive nuclear forces, the protons are held together inside a nucleus. These attractive forces overcome the electrostatic repulsions between the protons. See Fig. 13.4 on page 13.7. The negative P.E. signifies that the nuclear force is attractive in nature.
9. The strong attractive forces acting between the protons and neutrons of a nucleus which keep them bound together inside the tiny nucleus are called nuclear forces. The important properties of nuclear forces are as follows :

(i) Nuclear forces are the strongest attractive forces known in nature $F_G : F_E : F_N = 1 : 10^{36} : 10^{38}$.

(ii) They are short range forces effective upto 2 – 3 fermi from a nucleon.

(iii) They have charge independent nature.

(iv) They show saturation effect *i.e.*, a nucleon can interact only with a neighbouring nucleon.

(v) They are non-central forces *i.e.*, they do not act along the line joining the centres of the two nucleons.

10. (i) Saturation effect of nuclear forces.

(ii) Refer answer to Q. 6. on page 13.4.

11. (a) The graph showing the variation of P.E. of a pair of nucleons as a function of the separation r is shown in Fig. 13.4 on page 13.7.

(i) For $r < r_0$, nuclear force is repulsive.

(ii) For $r > r_0$, nuclear force is attractive.

(b) Refer answer to Q. 9 above.

12. Refer answer to Q.10. on page 13.8. When a nucleus is formed from its nucleons, some of their mass is converted into energy which binds the nucleons together inside the nucleus. This energy is called binding energy which is equivalent of mass defect.
13. Refer answer to Q. 11 on page 13.8.
14. Refer answer to Q. 13 on page 13.9.
15. See Fig. 13.5. Binding energy per nucleon of lighter nuclei is small. In an attempt to have higher B.E./A, lighter nuclei undergo nuclear fusion.
16. See Fig. 13.5. The binding energy per nucleon for nuclei with high mass numbers is low due to the large coulomb repulsion between the large number of protons present in these nuclei.
17. See Fig. 13.5. Refer answer to Q. 13 on page 13.9. A typical nuclear fission in which large amount of energy is released is as follows :

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n + Q$$

18. See Fig. 13.5 on Page 13.9.
19. (a) B.E. $= [Z_{m_H} + (A - Z)m_n - m({}^{A}_{Z}X)]c^2$

(b) See. Fig. 13.5 on page 13.9

From the binding energy per nucleon curve, it is clear that binding energy per nucleon of the fused nuclei is more than those of the light nuclei taking part in nuclear fusion. Hence energy gets released in the process.

20. Refer answer to Q. 20 on page 13.15.
21. Refer answer to Q. 25 on page 13.19.
22. Refer answer to Q. 28 on page 13.20.
23. Refer answer to Q. 29 on page 13.22.
24. Refer to points 28 and 29 of Glimpses on page 13.79.
25. Refer to points 27 and 29 of Glimpses on page 13.79.
26. Refer answer to Q. 22 on page 13.17.
27. Refer answer to Q. 17 on page 13.14.

28. Refer answer to Q. 21 on page 13.16.
29. Refer answer to Q. 23 on page 13.18.
30. Refer answer to Q. 21 on page 13.16.
31. τ represents mean life of a radioactive sample. Refer to point 29 of Glimpses on page 13.72.
32. Refer answer to Q. 21 on page 13.16 and see Fig. 13.9.
33. Refer answer to Q. 24 on page 13.18. See Problem 6 on page 13.55
34. Proceed as in Problem 7 on page 13.55.

$$R_1 : R_2 = 3 : 16.$$

35. (*a*) See Fig. 13.12 on page 13.22.
    (*b*) The plot for the distribution of kinetic energy of β-particles is shown in Fig. 13.11. Refer to the solution of Problem 22 on page 13.44.
36. (*a*) Refer answer to Q. 21 on page 13.16.
    (*b*) (*i*) $^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + e^+ + \nu$. The basic process underlying this β^+ decay is $p \rightarrow n + e^+ + \nu$.
    (*ii*) $^{22}_{10}$Ne is an *isobar* of $^{22}_{11}$Na as both have same mass number but different atomic number.
37. Refer answer to Q. 26 on page 13.19.
38. Refer answer to Q. 28 on page 13.20.
39. Refer answer to Q. 36 on page 13.31.
40. Refer answer to Q. 38 on page 13.32.
41. Refer answer to Q. 39 on page 13.33.
42. Refer answer to Q. 42 on page 13.34.
43. Refer answer to Q. 40 on page 13.33.
44. Refer answer to Q. 45 on page 13.36.
45. Refer answers to Q. 40 and Q. 46 above.
46. Refer answer to Q. 46 on page 13.36.
47. Refer answer to Q. 39 on page 13.33.
48. Refer answer to Q. 47 on page 13.36.
49. Refer answer to Q. 50 on page 13.42.
50. $$^{180}_{72}A \xrightarrow[{}^{4}_{2}\text{He}]{\alpha} {}^{176}_{70}A \xrightarrow[{}^{0}_{-1}e]{\beta} {}^{176}_{71}A_2 \xrightarrow[{}^{4}_{2}\text{He}]{\alpha} {}^{172}_{69}A_3 \xrightarrow{\gamma} {}^{172}_{69}A_4$$

    Mass number of A_4 = **172**.

    Atomic number of A_4 = **69**.
51. (*a*) Refer to the solution of Problem 23 on page 13.50.
    (*b*) See Fig. 13.20 on page 13.53.
52. (*a*) $^{10}_{5}\text{B} + {}^{1}_{0}\text{n} \longrightarrow {}^{4}_{2}\text{He} + {}^{7}_{3}\text{Li}$
    (*b*) $^{94}_{42}\text{Mo} + {}^{2}_{1}\text{H} \longrightarrow {}^{95}_{43}\text{Te} + {}^{1}_{0}\text{n}$
53. (*a*) (*i*) $^{208}_{84}\text{Po} \rightarrow {}^{204}_{82}\text{Pb} + {}^{4}_{2}\text{He} + \text{Q}$
    (*ii*) $^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + {}^{0}_{-1}e + \bar{\nu}$
    (*b*) The basic processes involved are :
    (*i*) β^- decay : $^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e + \bar{\nu}$
    (*ii*) β^+ decay : $^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{1}e + \nu$
54. $^{11}_{6}\text{C} \rightarrow {}^{11}_{5}X + e^+ + \nu + Q$

    $^{11}_{5}X$ is an isobar of $^{11}_{6}$C as both have same mass number.

    Mass defect, $\Delta m = m_N({}^{11}_{6}\text{C}) - m_N({}^{11}_{5}B) - m_e$

    In terms of atomic masses, we can write

    $$\Delta m = [m({}^{11}_{6}\text{C}) - 6m_e] - [m({}^{11}_{6}B) - 5m_e] - m_e$$
    $$= m({}^{11}_{6}\text{C}) - m({}^{11}_{6}B) - 2m_e$$

    Ignoring the mass of electrons, we get

    $$\Delta m = m({}^{11}_{6}\text{C}) - m({}^{11}_{6}B)$$
    $$= (11.011434 - 11.009305)u$$
    $$= 0.002129 \text{ u.}$$
    $$Q = \Delta m \times 931.5 \text{ MeV}$$
    $$= 0.002129 \times 931.5 \text{ MeV}$$
    $$= \mathbf{1.98\ MeV}.$$
55. The breaking of a heavy nucleus into smaller nuclei is called nuclear fission while the combining of two light nuclei to form a heavy nucleus is called nuclear fusion.

    In both the processes, the binding energy of the daughter nuclei is more than that of the parent nuclei. The difference in binding energy is released in the form of energy. In both processes, some mass gets converted into energy.

    Numerical. Energy released

    $$= [m({}^{2}_{1}\text{H}) + m({}^{3}_{1}\text{H}) - m({}^{4}_{2}\text{He}) - m(n)] \times 931.5 \text{ MeV}$$
    $$= [2.014102 + 3.016049 - 4.002603 - 1.008665] \times 931.5 \text{ MeV}$$
    $$= 0.018883 \times 931.5 \text{ MeV} = \mathbf{17.59\ MeV.}$$

TYPE C : LONG ANSWER QUESTIONS (5 marks each)

1. What do you mean by binding energy of a nucleus ? Obtain an expression for binding energy. How binding energy per nucleon explains the stability of nucleus ? [Punjab 03 C]
2. Define the term binding energy. The binding energy of a nucleus $^{A}_{Z}X$ is given by the formula :

$$\text{B.E.} = [Zm_H + (A - Z)\, m_n - m({}^{A}_{Z}X)]\, c^2$$

where $m(^A_Z X)$ is the atomic mass of X. Derive this equation, state clearly the approximation involved and say it is very safe approximation.

3. (*a*) Draw the plot of binding energy per nucleon (BE/A) as a function of mass number A. Write two important conclusions that can be drawn regarding the nature of nuclear force.
   (*b*) Use this graph to explain the release of energy in both the processes of nuclear fusion and fission.
   (*c*) Write the basic nuclear process of neutron undergoing β-decay. Why is the detection of neutrinos found very difficult ? [CBSE OD 13]
4. What is radioactivity ? State the laws of radioactive decay. Show that radioactive decay is exponential in nature. [Punjab 2000, 02, 04]
5. Use the basic law of radioactive decay, to show that radioactive nuclei follow an exponential decay law. Hence obtain a formula, for the half-life of a radioactive nuclide, in terms of its disintegration constant. [CBSE D 04C]
6. What do you understand by radioactivity and half-life ? Plot an accurate graph to show how the number of radioactive atoms of a give element (expressed as percentage of those initially present) varies with time. Use the time scale extending over five half-lives. [ISCE 90]
7. What is α-decay ? Discuss briefly by using tunneling effect. Show that the kinetic energy of an α-particle is
$$K_{He} = \frac{m_Y}{m_Y + m_{He}} . Q$$
where the symbols have their usual meanings. [Punjab 04C ; Himachal 06]
8. What is β-decay ? How can radioactive nuclei emit β-particles even though nuclei do not contain these particles. Hence explain why the mass number of a radioactive nucleus does not change during β-decay. Why is the energy distribution of β-rays continuous ? [CBSE D 04C]
9. What is a chain reaction ? Explain the difficulties encountered in sustaining a chain reaction.
10. State the principle involved in a nuclear reactor. Draw a labelled diagram of a nuclear reactor and explain the functions of moderator, control rods and coolant in it. [Haryana 10,11]
11. What is nuclear fusion ? Explain, how such a large amount of energy is produced inside the sun through proton-proton cycle and carbon-carbon cycle. [Punjab 03C]
12. Differentiate between nuclear fission and nuclear fusion. Which one of these processes produces energy (*i*) in nuclear reactor and (*ii*) in the sun ?

Answers

1. Refer answer to Q. 12 on page 13.8.
2. Refer answer to Q. 12 on page 13.8.
3. (*a*) For binding energy curve, see Fig. 13.5 on page 13.9. The constancy of binding energy per nucleon over a wide range of mass number $30 < A < 170$ indicates that
   (*i*) Nuclear force is a short range force.
   (*ii*) Nuclear force shows saturation effect.
   (*b*) Refer answer to Q. 13 on page 13.9.
   (*c*) The β-decay of a neutron can be represented as follows :
$$^1_0 n \rightarrow ^1_1 H + ^0_{-1} e + \overline{\nu} + Q$$
   Neutrinos interact very weakly with matter. So they have a very high penetrating power. Hence the detection of neutrinos is very difficult.
4. Refer answer to Q. 20 on page 13.15.
5. Refer answer to Q. 20, 22 on pages 13.15 and 13.17.
6. See Fig. 13.9 on page 13.17.
7. Refer answer to Q. 26, 27 on pages 13.19 and 13.20.
8. Refer answer to Q. 28 on page 13.20.
9. Refer answer to Q. 39, 41 on pages 13.33 and 13.34.
10. Refer answer to Q. 43 on page 13.35.
11. Refer answer to Q. 47 on page 13.36.
12. Refer answer to Q. 49 on page 13.42.

TYPE D : VALUE BASED QUESTIONS (4 marks each)

1. Anuj's mother was having a constant headache and was diagnosed with tumour. She was avoiding treatment because of financial constraints. When Anuj learnt about it, he cancelled his plans to go abroad and decided to use that money for the treatment and care of his mother. Answer the following questions.
   (*a*) What, according to you, are the values displayed by Anuj ?
   (*b*) Which type of radiation do you think could be used for the treatment ?
   (*c*) When are γ-rays emitted by a nucleus ? [CBSE OD 13C]

2. Deepti's uncle, who was a junk-dealer, was getting weak day by day. His nails were getting blue, he started losing his hair. This happened immediately after he purchased a big container of heavy mass from the chemistry department of a national university. Doctors advised him hospitalization and suspected he had been exposed to radiation. Her uncle didn't know much about radiations but Deepti immediately convinced her uncle to get admitted and start treatment.
   (*a*) What according to you are the values utilized by Deepti to convince her uncle to get admitted in hospital ?
   (*b*) Name the radioactive radiations emitted by a radioactive element.
3. Kajol's grandfather was reading an article in a newspaper. He read that after so many years of atomic bombing in Hiroshima or Nagasaki, Japan National Census indicated that children born even now are genetically deformed. Her grandfather was not able to understand the reason behind it. He asked his granddaughter Kajol, who was a science student of Class XII. Kajol sat with her grandfather and showed him pictures from some books and explained the harmful effects of radiations.
   (*a*) What are the values/skills utilized by Kajol to make her grandfather understand the reason of genetically deformity ?
   (*b*) Name the nuclear reactions that occur in an atom bomb.
4. Ramaswami, a resident of Kundakulam, was all set to leave everything and shift to another place in view of the decision of Government to start nuclear thermal power plant at Kundakulam. His grand-daughter Manika, a science student, was really upset on the ignorant decision of her grandfather. She could finally convince him not to shift, since adequate safety measures to avoid any nuclear mishap have already been taken by the Government before starting nuclear thermal plants.
   (*a*) What is the value displayed by Manika in convincing her grandfather ?
   (*b*) Name the main components of a nuclear reactor.
   (*c*) Name the working principle of a nuclear reactor.
   (*d*) Why is heavy water used as a moderator ?
5. For the past some time, Aarti had been observing some erratic body movement, unsteadiness and lack of coordination in the activities of her sister Radha, who also used to complain of severe headache occasionally. Aarti suggested to her parents to get a medical check-up of Radha. The doctor thoroughly examined Radha and diagnosed that she has a brain tumour.
   (*a*) What, according to you, are the values displayed by Aarti ?
   (*b*) How can radioisotopes help a doctor to diagnose brain tumour ? [CBSE OD 14]

Answers

1. (*a*) Affection, care and concern. (*b*) γ-rays.
   (*c*) After losing an α- or β-particle, a daughter nucleus is left in the excited state. It comes to its ground state by emitting one or more γ-ray photons.
2. (*a*) Scientific awareness, critical thinking and decision making. (*b*) α-, β- and γ-rays.
3. (*a*) Sympathy and compassion.
   (*b*) Nuclear fission reactions.
4. (*a*) Awareness and social responsibility.
   (*b*) Nuclear fuel, moderator, control rods, coolant and outer shielding.
   (*c*) Controlled chain reaction.
   (*d*) Heavy water contains protons. Fast moving neutrons undergo elastic collisions with these slow moving protons and thus get slowed down.
5. (*a*) Keen observer and affection/care/concern for her sister.
   (*b*) A small amount of radioisotope like radio-iodine is injected into the body alongwith organic dyes which are absorbed strongly by the tumour tissues than the normal tissues. By detecting the emitted radiation, the radiologists get information about the size and location of the tumour.

Nuclei

GLIMPSES

1. Nucleons. Protons and neutrons which are present in the nuclei of the atoms are collectively known as nucleons.

2. Atomic Number. The number of protons present in the nucleus is called the atomic number. It is denoted by Z.

3. Mass number. The total number of protons and neutrons present in a nucleus is called the mass number of the element. It is denoted by *A*.

   Number of protons in an atom $= Z$

   Number of electrons in an atom $= Z$

   Number of nucleons in an atom $= A$

   Number of neutrons in an atom $= N = A - Z$.

4. Nuclear mass. The total mass of the protons and neutrons present in a nucleus is called the nuclear mass.

5. Nuclide. A nuclide is a specific nucleus of an atom characterised by its atomic number *Z* and mass number *A*. It is represented as

$$ {}_Z^A X $$

   X = chemical symbol of the element,

   Z = atomic number, and

   A = mass number.

6. Isotopes. The atoms of an element, which have the same atomic number but different mass number are called isotopes.

7. Isobars. The atoms of different elements (different atomic number) but having the same mass number are called isobars.

8. Isotones. The nuclides having the same number of neutrons are called isotones.

9. Isomers. These are the nuclei with same atomic number and same mass number but in different energy states.

10. Atomic mass unit (amu). It is defined as $\frac{1}{12}$ th of the mass of one ${}_6^{12}C$ atom. Its value is given by

$$1 \text{ amu} = 1.660565 \times 10^{-27} \text{ kg} = 931 \text{ MeV}.$$

11. Electron volt. It is defined as the energy acquired by an electron when it is accelerated through a potential difference of 1 volt and is denoted by eV.

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J},$$
$$1 \text{ MeV} = 10^6 \text{ eV} = 1.602 \times 10^{-13} \text{ J}.$$

12. Relation between size of nucleus and mass number. It is found that the radius *r* of a nucleus is proportional to the cube root of its mass number.

$$r = r_0 A^{1/3},$$

   where $r_0 = 1.2 \times 10^{-15}$ m
   $= 1.2$ fermi for electrons as probes.

13. Nuclear density. The density of a nucleus is independent of the size of the nucleus and is given by

$$\rho_{nu} = \frac{\text{Nuclear mass}}{\text{Nuclear volume}} = \frac{m_{nu}}{\frac{4}{3}\pi R^3}.$$

   The nuclear density is of the order of 10^{17} kg m^{-3}.

14. Discovery of neutrons. Neutrons were discovered by Chadwick in 1932. When beryllium nuclei are bombarded by α-particles, highly penetrating radiations are emitted, which consist of neutral particles, each having mass

nearly that of a proton. These particles were called neutrons.

$$^{4}_{2}\text{He} + ^{9}_{4}\text{Be} \longrightarrow ^{1}_{0}n + ^{12}_{0}\text{C}$$

A free neutron decays spontaneously, with a half life of about 1000 s, into a proton, electron and an antineutrino.

$$^{1}_{0}n \longrightarrow ^{1}_{1}\text{H} + ^{0}_{-1}e + \bar{\nu}.$$

15. Nuclear forces. These are the strong attractive forces which hold protons and neutrons together in a tiny nucleus. These are short range forces which operate over very short distances of about 2 – 3 fm from a nucleon. The nuclear force does not depend on the charge of the nucleus.

16. Mass defect. The difference between the rest mass of a nucleus and the sum of the rest masses of its constituent nucleons is called its mass defect. It is given by

$$\Delta m = Zm_p + (A - Z)\, m_n - m.$$

17. Binding energy. It may be defined as the energy required to break up a nucleus into its constituent protons and neutrons and to separate them to such a large distance that they may not interact with each other.

It may also be defined as the surplus energy which the nucleons give up by virtue of their attractions when they become bound together to form a nucleus.

The binding energy of a nucleus $^{A}_{Z}X$ is given by

$$\Delta E_b = [Z\, m_H + (A - Z)\, m_n - m]\, c^2.$$

18. Binding energy per nucleon. It is the average energy required to extract one nucleon from the nucleus. It is obtained by dividing the binding energy of a nucleus by its mass number.

$$\Delta E_{bn} \text{ or } \bar{B} = \frac{\text{B.E.}}{A} = \frac{[Zm_H + (A - Z)\, m_n - m]\, c^2}{A}$$

In the mass number range $A = 30$ to 170, the binding energy per nucleon is nearly constant, about 8 MeV per nucleon.

19. Packing fraction. The packing fraction of a nucleus is its mass defect per nucleon.

$$\text{P.F. of a nucleus} = \frac{\text{Mass defect}}{\text{Mass number}} = \frac{\Delta m}{A}.$$

It is directly related to the availability of nuclear energy and the stability of the nucleus.

20. Nuclear energy levels. Like an atom, the nucleus exists in nucleonic configurations, which correspond to nuclear stationary states. The stationary state of the lowest energy is called the ground state. When a nucleus makes a transition from one level to a level of lower energy, the emitted photon belongs to the gamma ray region of the electromagnetic spectrum.

21. Radioactivity. It is the phenomenon of spontaneous disintegration of the nucleus of an atom with the emission of one or more radiations like α-particles, β-particles or γ-rays. The substances which spontaneously emit these penetrating radiations are called radioactive substances.

22. Soddy-Fajan's displacement law. It states that

(*i*) When a radioactive nucleus emits an α-particle, its atomic number decreases by 2 and mass number decreases by 4.

(*ii*) When a radioactive nucleus emits β-particle, its atomic number increases by 1 but mass number remains same.

(*iii*) The emission of a γ-particle does not change the mass number or the atomic number of the radioactive nucleus.

23. Alpha decay. It is the process of emission of an α-particle from a radioactive nucleus.

$$^{A}_{Z}X \longrightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}\text{He} + Q.$$

24. Beta decay. It is the process of emission of an electron or a positron from a radioactive nucleus.

$$^{A}_{Z}X \longrightarrow {}_{Z+1}^{A}Y + \beta^- + \bar{\nu} \qquad (\beta^-\text{-decay})$$

$$^{A}_{Z}X \longrightarrow {}_{Z-1}^{A}Y + \beta^+ + \nu \qquad (\beta^+\text{-decay})$$

25. Gamma decay. It is the process of emission of a γ-ray photon during the radioactive disintegration of a nucleus. This occurs when a nucleus in an excited state makes a transition to a state of lower energy

$$^{A}_{Z}X \longrightarrow ^{A}_{Z}X + \gamma$$

(Excited state) (Ground state)

26. Radioactive decay law. The number of atoms of a radioactive sample disintegrating per second at any instant is directly proportional to the number of undecayed radioactive nuclei present at that instant.

$$\frac{dN}{dt} = -\lambda N$$

The law may also be expressed as $N = N_0 e^{-\lambda t}$ where N_0 is the number of nuclei at $t = 0$ and λ is decay constant.

27. Decay constant. It may be defined as the reciprocal of the time interval in which the number of active nuclei in a given radioactive sample reduces to 36.8% (or $1/e$ times) of its initial value. Its units are s^{-1}, day^{-1}, $year^{-1}$, etc.

28. Half-life. The half-life of a radioactive substance is the time in which one-half of the initial number of nuclei disintegrates.

$$T_{1/2} = \frac{0.693}{\lambda} \; ; \quad N = N_0\left(\frac{1}{2}\right)^n$$

where n = number of half-lives in time $t = \frac{t}{T_{1/2}}$.

Its units are s, day, year, etc.

29. Mean-life. It may be defined as the ratio of the combined age of all the atoms to the total number of atoms present in the given sample.

$$\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44\, T_{1/2}.$$

Its units are s, day, year, etc.

30. Decay rate or activity of a sample. It is the number of radioactive disintegrations taking place per second in a given sample.

$$R = \left|\frac{dN}{dt}\right| = \lambda N = \lambda N_0 e^{-\lambda t} \quad \text{or} \quad R = R_0 e^{-\lambda t}.$$

31. Becquerel. It is the SI unit of activity and is defined as the decay rate of one disintegration per second.

1 bequerel = 1 bq = 1 decay per second.

32. Curie. One curie is the decay rate of 3.7×10^{10} disintegrations per second.

1 Ci (curie) = 3.70×10^{10} disintegrations / s.

33. Rutherford. One rutherford is the decay rate of 10^6 disintegrations per second.

1 rd (rutherford) = 10^6 disintegrations/s.

34. Natural radioactivity. It is the phenomenon of the spontaneous emission of α-, β- or γ-radiations from the nuclei of naturally occurring isotopes.

35. Artificial or induced radioactivity. It is the phenomenon of inducing radioactivity in certain stable nuclei by bombarding them by suitable high energy particles.

36. Nuclear reaction. It is a reaction which involves the change of stable nucleus of one element into the nucleus of another element, by bombarding the former with suitable high energy particles.

37. Nuclear fission. It is the process in which a heavy nucleus ($A > 230$) when excited gets split up into two smaller nuclei of nearly comparable masses. For example,

$${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n + Q.$$

38. Thermal neutrons. These are the slow moving neutrons of energy 0.0253 eV, corresponding to the velocities of 2200 ms^{-1}.

39. Multiplication factor. The multiplication factor of a fissionable mass is defined as the ratio of the number of neutrons present at the beginning of a particular generation to the number of neutrons present at the beginning of the previous generation.

If $k > 1$, the chain reaction *grows*.

If $k = 1$, the chain reaction remains *steady*.

If $k < 1$, the chain reaction gradually *dies out*.

40. Critical size and critical mass. The size of the fissionable material for which multiplication factor is unity is called critical size and its mass is called critical mass of the material. The chain reaction in this case remains steady or sustained.

41. Moderator. Any substance which is used to slow down fast moving neutrons to thermal energies is called a moderator. The commonly used moderators are water, heavy water (D_2O) and graphite.

42. Nuclear reactor. It is a device in which a nuclear chain reaction is initiated, maintained and controlled. The reaction is controlled by using neutron absorbing materials like cadmium rods.

43. Nuclear fusion. It is the process of fusion of two smaller nuclei into a heavier nucleus with the liberation of large amount of energy. For example,

$${}^{2}_{1}H + {}^{2}_{1}H \longrightarrow {}^{4}_{2}He + 24\text{ MeV}.$$

These reactions require the extreme conditions of temperature and pressure so that the reacting nuclei can overcome their electrostatic repulsion. For this reason, these reactions are called *thermonuclear reactions*.